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A preliminary error analysis of the gravity field recovery from a lunar Satellite-to-Satellite mission

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Summary. A low cost lunar Satellite-to-Satellite radio tracking mission in a low-low configuration could considerably improve the existing knowledge about the lunar gravity field. The impact of various mission parameters that may contribute to the recovery of the gravity field, such as satellite altitude, satellite separation, mission duration, measurement precision and sampling interval were quantified using the Jekeli-Rapp algorithm. Preliminary results indicate that the gravity field resolution up to harmonic degree 40 to 80 is feasible depending on various mission configurations. Radio tracking data from a six-month mission with a precision of 1 mm s^{-1} every 10 s and 300 km satellite separation at 150 km altitude will permit the determination of $5^o x 5^o$ mean gravity anomalies with an error of approximately 15 mgals. Consideration of other unaccounted error sources of instrumental, operational as well as environmental nature may lower this resolution.

Introduction

Next generation lunar missions (Synthesis Group on America's Space Exploration Initiative 1991) call for improved gravity field resolution. Improved determination of the gravity field is needed for accurate navigational requirement in the lunar environment. Improved resolution implies more accurate orbit determination for other science missions such as lunar altimetry. Improved gravity field together with lunar altimetry contributes to accurate inferences on the mass distribution inside the moon which provides constraints on its evolution (Bills 1991).

A possible mission scenario involves a spacecraft in a low, circular, lunar polar orbit tracking a small light-weight passive sub-satellite in the same orbit with a specified separation distance from the main spacecraft. Since small satellites require a moderate launch vehicle, the cost of such a mission would be relatively low and deployment of low cost Doppler technology for tracking the passive satellite by the active one will provide in-situ, satellite-borne and continuous measurements.

Although there exist some information about the long-wavelength portion of the lunar gravity (Liu and Laing, 1971; Ferrari and Ananda, 1977; Bills and Ferrari, 1980—a 16x16 harmonic field model; Sagitow et al., 1986) they all suffer from the lack of information from the far side of the moon where the satellite escapes from direct tracking from earth. Therefore, an important contribution of a lunar satellite-to-satellite tracking mission would be to provide direct gravity information about the far side of the moon for the first time. These measurements can be reinforced, for the long-wavelength portion of the gravity field information, by tracking one of the satellites from the Earth.

In this study, the impact of various mission parameter that may contribute to the recovery of the gravity field from Satellite-to-Satellite (SST) Doppler measurements such as; satellite altitude, satellite separation, mission duration, measurement precision and sampling interval were quantified using the Jekeli-Rapp algorithm. Next section summarizes the algorithm followed by the section on 'Numerical Results and Conclusion'.

The Gravity Field Recovery Error Analysis Algorithm

The approach used for the error analysis of the lunar gravity field recovery is the Jekeli-Rapp algorithm (1980) developed for an earth gravity mapping satellite mission (GRAVSAT). Its ease of use makes the algorithm very attractive for preliminary computations in which the impact of various mission parameters can rapidly be assessed. The algorithm, in the past, has been found to produce reasonable results against other approaches (Colombo 1981, Jekeli and Rapp 1980). This section outlines the algorithm. Complete derivations can be found in Jekeli and Rapp (1980).

If an observed quantity F can be expressed as a series of spherical harmonic functions at the satellite altitude as,

$$F(\theta,\lambda)=\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\bar{f}_{nm}\bar{Y}_{nm}(\theta,\lambda) \quad |m|\le n,\ n\ge 2 \quad (1)$$

where (θ,λ) are spherical coordinates and the $\bar{Y}_{nm}$ are fully normalized harmonic functions where n represents the harmonic degree and m is the harmonic order, then the error spectrum of this quantity can be shown as,

$$m(\bar{f}_{nm})=\sqrt{\frac{\Delta\sigma}{4\pi}}\ m(F) \quad (2)$$

where $\Delta\sigma$ is the block size in units of square radians. In arriving at this relationship it is assumed that the observations are uncorrelated and uniformly distributed with variance $m^2(F)$. All measurements are made in blocks of equal size which uniformly cover the entire sphere and as a result requiring polar satellites.

Now consider two satellites which are in a low-low configuration with one satellite Doppler tracking the other. A single Doppler measurement in this configuration represents the velocity difference of two satellites, i.e.,

$$\Delta V_H = V_P - V_Q \quad (3)$$

where V_P and V_Q are the velocities of the tracking and the leading satellite at the moment of observation. The root mean square velocity difference denoted by $\Delta\bar{V}_H$, over all possible directions is a harmonic function and can be expressed in a series of harmonic functions. The spectrum of $\Delta\bar{V}_H$ may be expressed as,

$$\sigma_{nm}(\Delta\bar{V}_H)=\sqrt{\frac{r}{\gamma}}\frac{1}{n-1}\sqrt{\frac{2(1-P_n(\cos\psi_{PQ})c_n}{2n+1}}\left(\frac{R}{r}\right)^{n+1} \quad |m|\le n,\ n\ge 2 \quad (4)$$

In the above expression R is the mean lunar radius, r is the magnitude of the selenocentric position vector of the satellite, Ψ_{PQ} is the angular satellite separation, γ denotes the lunar normal gravity, c_n are the gravity anomaly degree variances, and P_n represents the Legendre polynomials. In deriving (4), it is assumed that the velocity signal is isotropic and homogeneous on the sphere, and that the spectrum of horizontal velocity difference in any direction is representative for all directions.

Comparison of the signal spectrum given by (4) against the measurement error spectrum given by (2) gives the maximum resolution: n_{max}. The resolution of the data can be decided such as the frequency at which signal to noise ratio is one. Maximum resolution of data, in turn, warrants the maximum resolution of the estimated quantities from the data such as gravity anomalies and geoid undulations. Since the estimated quantities are limited in resolution, the corresponding error estimates need to account for an omission error along with a commission error which is a result of the data uncertainties. In the case of estimation of mean quantities, the omission error is a result of averaging over a block (spherical cap), thereby dampening the magnitudes of the high frequency components.

Again, without derivation, the total error of mean gravity anomalies $m_{TOT}(\Delta\bar{g})$ for a prescribed block size, defined as the root sum square of the commission error $m_c(\Delta\bar{g})$ and the omission error $m_o(\Delta\bar{g})$, is given by

$$m_{TOT}^2(\Delta\bar{g}):=m_c^2(\Delta\bar{g})+m_o^2(\Delta\bar{g}) \quad (5)$$

with

$$m_c^2(\Delta\bar{g}):=\frac{\gamma}{r}\frac{\Delta\sigma}{4\pi}m^2(\Delta v_H)\sum_{n=2}^{n_{max}}\frac{\beta_n^2(n-1)^2(2n+1)}{2(1-P_n(\cos\psi_{PQ}))}\left(\frac{r^2}{R^2}\right)^{n+1} \quad (6)$$

$$m_o^2(\Delta\bar{g}):=\sum_{n=n_{max}+1}^{\infty}\beta_n^2 c_n \quad (7)$$

The averaging (smoothing) coefficients β_n are given recursively by

$$\beta_n=\frac{2n-1}{n+1}\cos\psi_0\beta_{n-1}-\frac{n-2}{n+1}\beta_{n-2} \quad (8)$$

$$\beta_0=1 \quad \beta_1=\frac{(1+\cos\psi_0)}{2} \quad (9)$$

where ψ_0 is the radius of the cap having the same area of as the block. Inherent in these equations is the assumption that the spectral components of the $\Delta\bar{V}_H$ are directly proportional to those of the disturbing potential.

Making use of the well-known Brun's formula which relates the distubing potential to geoid undulations, similar relationships are obtained for the geoid undulation error estimates

$$m_{TOT}^2(\bar{N}):=m_c^2(\bar{N})+m_o^2(\bar{N}) \quad (10)$$

with

$$m_c^2(\bar{N}):=\frac{R^2}{\gamma r}\frac{\Delta\sigma}{4\pi}m^2(\Delta v_H)\sum_{n=2}^{n_{max}}\frac{\beta_n^2(2n+1)}{2(1-P_n(\cos\psi_{PQ}))}\left(\frac{r^2}{R^2}\right)^{n+1} \quad (11)$$

$$m_o^2(\bar{N}):=\frac{R^2}{\gamma^2}\left[\sum_{n=n_{max}+1}^{\infty}\frac{\beta_n^2 c_n}{(n-1)^2}\right] \quad (12)$$

Once the maximum resolution is determined, equations (5) - (12) can be evaluated to asses the omission and the commission errors of the mean anomaly and geoid undulation recoveries. A battery of results based on different mission and observation parameters are given in the following section.

Numerical Results and Conclusion

Alternative mission parameters are summarized in Table 1. Anomaly degree variance model is based on scaled Kaula's rule (by a factor of 35) for the lunar environment (Bills and Ferrari 1980). The model may not be very realistic at higher degrees as in the case of earth gravity field representations but appropriate when no other information is available about the power of the gravity field. The maximum resolution will change considerably in the presence of more power. The error estimates for the mean anomalies, however, were found to be quite robust against these changes because of the averaging of the gravity field information over a block.

Table 1. Mission parameters

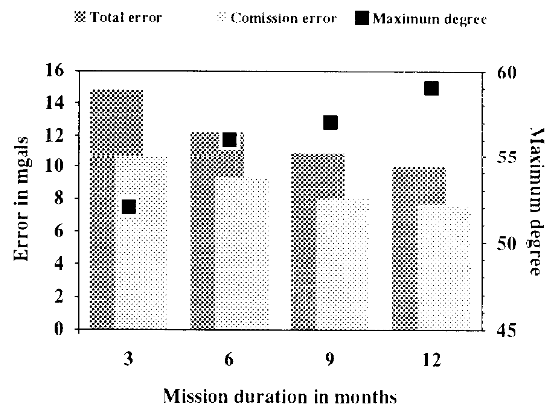
Variable mission parameters
Satellite Altitude: 200, 250, 300, 350, 400 km.
Satellite separation: 50, 100, 150 km.
Mission duration: 3, 6, 9 ,12 months.
Instrument precision: 0.01, 0.001, 0.0001 m s^{-1}
Sampling interval: 5, 10, 15, 20, 25 s.
Block Size: 1, 2, 5, 10 (degree x degree).

Constants
Anomaly degree variancemodel (Scaled Kaula's rule):
$c_n := [\gamma(n-1)\frac{3.5x10^{-4}}{n^2}]^2$
Selenocentric normal gravity: 1.624 m s^{-2}
Mean lunar radius: 1737530 m.

The influence of the mission duration on the recovery of the mean gravity anomalies is a result of increasing the number of observations thereby improving the resolution, and decreasing the errors due to random effects. Longer mission durations are necessary to generate sufficient number of measurements to homogeneously sample the lunar gravity field for an improved resolution. Regular sampling of the gravity field is also needed for the separability of harmonic coefficients.

Figure 1 shows that for a satellite separation of 300 km, Doppler precision of 1 mm/s every 10 s, and satellite altitude of 150 km, the resolution can reach up to 59 degree with a total error of 10 mgal for 5x5 degree block size based on a year of continuous tracking. During this period, approximately 3.1 million observations are generated. This number reduces to approximately 0.8 million for a 3 month mission duration. Nevertheless, it should be kept in mind that the improvements due to the increasing number of observations are usually limited by the presence of systematic and unmodeled effects on the satellites.

In figure 2 the error estimates are given as a function of the mean anomaly block sizes. The maximum resolution, in this scenario, is 56. Both satellites are assumed to be in a circular orbit of 150 km, with separation of 300 km. The mission duration is six months and the instrument precision is assumed to be 1 mm s^{-1} every 10 s. Considerable omission error reduction is observed, as expected, for the larger block size. Reduction in total error is nevertheless accompanied by the loss of resolution in the gravity anomaly information.



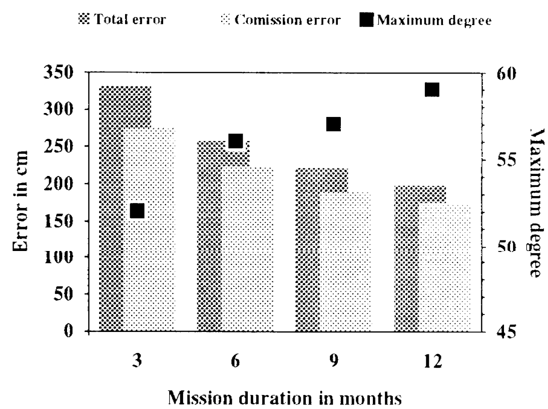


Fig.1. Satellite separation is 300 km, Precision is 1 mm/s every 10 s, satellite altitude is 150 km, 5x5 block size.

The steady increase in the total error as a function of increasing sampling interval, i.e., fewer observations and coarser resolution, can be traced in Figure 3 for satellite separation of 300 km at 150 km altitude and measurement precision of 1 mm s^{-1} for a six month mission. Figure 4, on the other hand, shows that marked improvements are possible for improved instrument accuracies. Although more frequent integration (averaging) together with more precision improves the resolution, they are unfortunately limited by uncontrolled systematic instrumental errors. Currently proposed technology, for instance, has a limit of 0.4 mm s^{-1} which occurs at 0.6 s averaging based on a standard Doppler measurement accuracy of 1 mm s^{-1}over 0.1 s^{-1} (Bills, 1992, private communication).

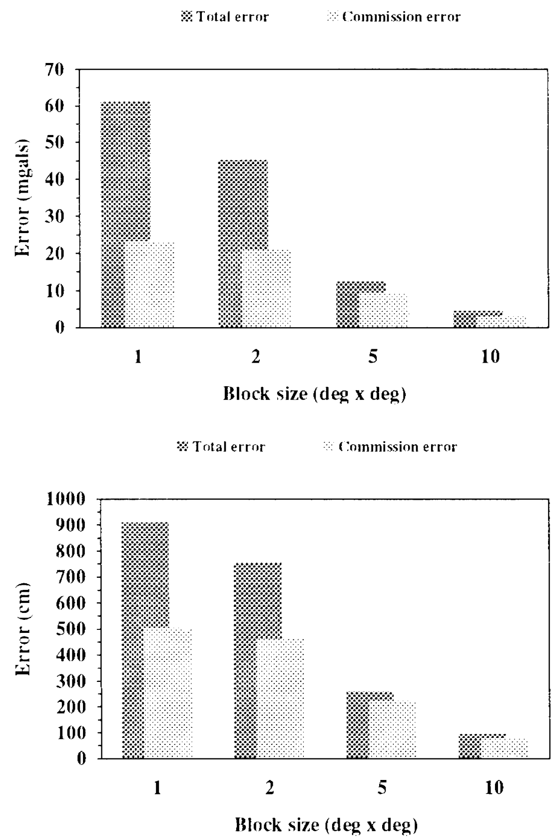
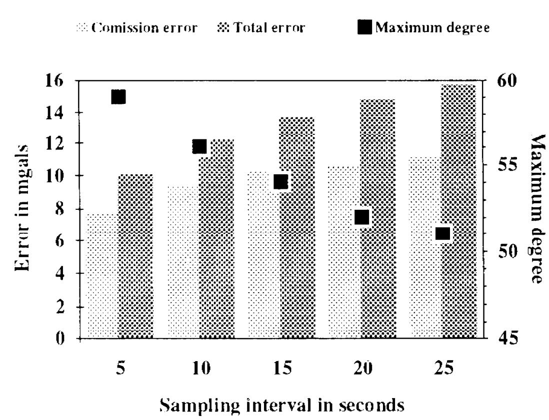


Fig.2. Maximum resolution is 56, satellite separation is 300 km, instrument precision is 1 mm/s every 10 s, and mission duration is 6 months. Both satellites are in low-low configuration at 150 km altitude.

A range of 200 to 400 km satellite altitudes was considered. Although, the satellite altitudes that are less than 200 km can improve the resolution, the presence of mass concentrations (mascons) may have undue influence on the satellite orbits, thereby, considerably shortening the mission duration. Satellite orbits larger than 400 km on the other hand, will contribute more to the long-wavelength portion of the gravity field recovery.



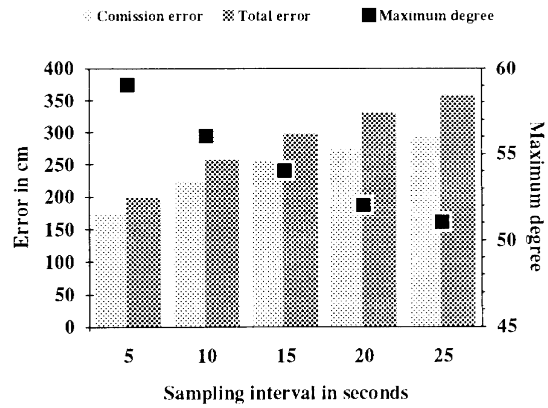


Fig.3. Satellite separation is 300 km, Precision 1 mm/s, satellite altitude 150 km, 5x5 block size, mission duration is 6 months

The variation of the resolution and the corresponding error estimates for the $5^{o}x5^{o}$ mean anomaly blocks are given in figure 5 for a satellite separation of 300 km, measurement frequency of 1 mm s^{-1} every 10 s, and for a six-month mission duration.

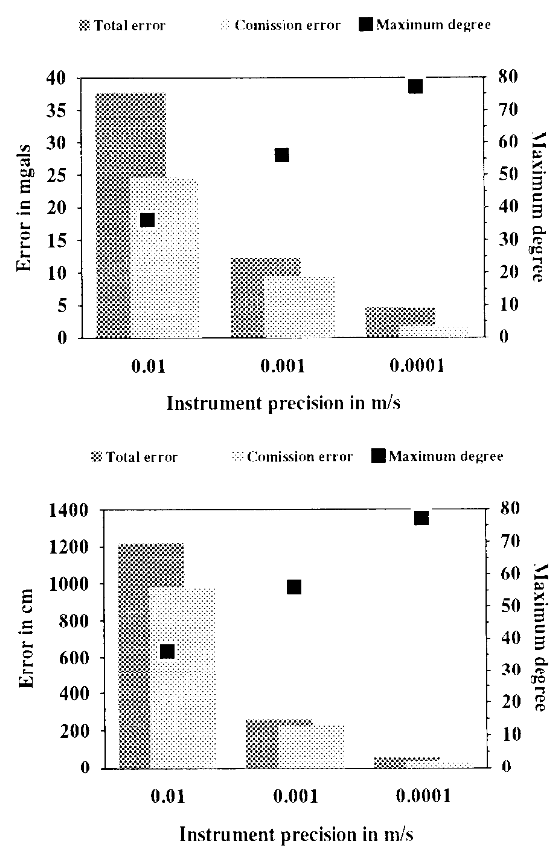


Fig.4. Satellite separation is 300 km, measurement frequency is every 10 s, satellite altitude 150 km, 5x5 block size, mission duration is 6 months

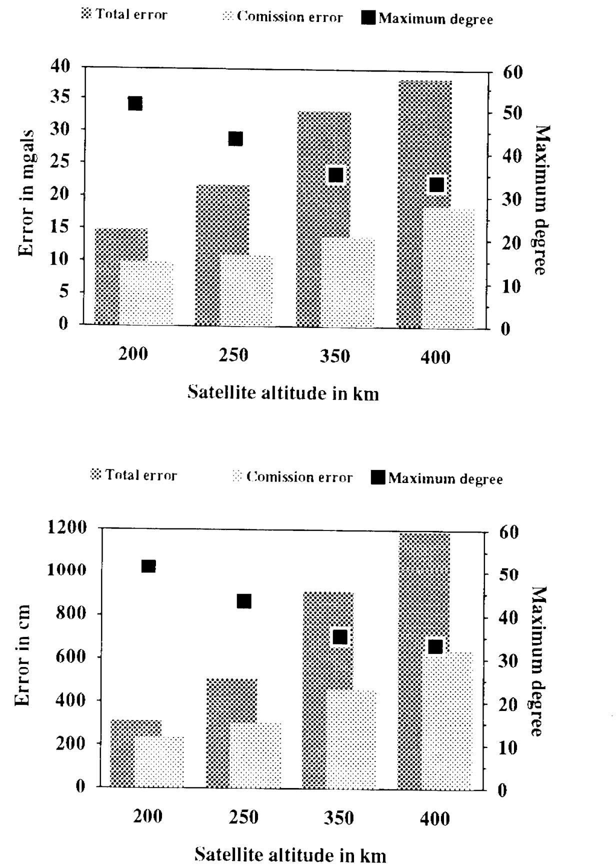


Fig.5. Satellite separation is 300 km, measurement frequency is 1 mm/s every 10 s, 5x5 block size, mission duration 6 months

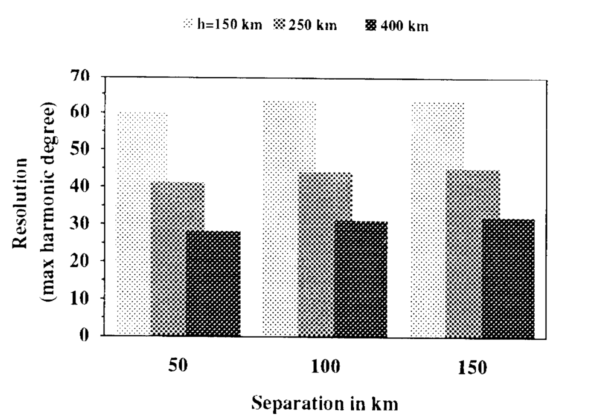


Fig.6. Instrument precision is 1 mm/s every 10 s; mission duration is 6 months; 5x5 block size is considered.

In figure 6 the impacts of various ranges of satellite separations at 150 km altitude were displayed for the same mission configuration as above. However, maintaining a fixed separation may be difficult in reality. Again, the lunar mascons may not influence one satellite orbit exactly the same way as the other satellite orbit because of the satellite hardware, orbit injections, and will impose frequent orbit maneuvers to accommodate to a fixed satellite separation at lower orbits. Fortunately, the results indicate that there is only minute variability in the errors for different satellite separations.

Overall, the above results indicate a wide range of resolutions (40 - 80) as a function of satellite altitude, instrument precision, frequency of measurements and mission duration. The error analysis approach was rudimentary in this study and the unaccounted error sources in the algorithm will decrease the resolution. However, the ranges of results for all the scenarios point to the possibility of marked improvements, through today's technology, in the existing knowledge about the lunar gravity field and are encouraging for further studies.

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