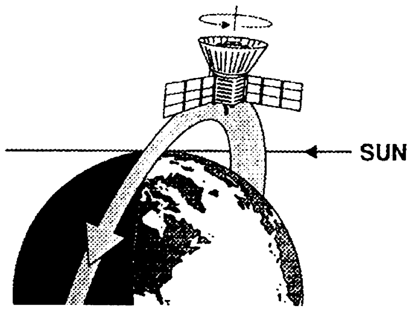

A QUICK LOOK INTO THE TWO DIMENSIONAL POWER SPECTRUM OF A PATCH ON THE SKY

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A QUICK LOOK INTO THE TWO DIMENSIONAL POWER SPECTRUM OF A PATCH ON THE SKY

Summary

A scheme has been setup to investigate two dimensional power spectra using DIRBE photometry. Tests were performed by doing the two dimensional power spectra of a selected patch on the celestial sphere for various bands using data sets of DIRBE photometry which are separated from one another by a week. These preliminary results are not very informative because of the noise level of the daily values, quiet background, and the missing data which adversely affect the analysis.

Introduction

The purpose of this study is to set up the rudimentary codewhich permit investigating the spatial two dimensional (2D) power spectrum content of the DIRBE photometry for sky patches. Comparison of 2D spectra of the same patch as a function of time will reveal information about the repeatability of the observations and thereby their precision in the frequency domain. The assumption made here is that the zodiacal light contribution imposes only low frequency components to the spectra.

As this document is a quick look into the constructionof 2D spectra, it only represents a computational exercise to gain insight about the process and the results should be taken with a big grain of salt.

Input data

This section reads and converts the input data for the 2D Complex Fast Fourier Transform (CFFT). You may skip to the next section if you are not interested in the conversion process.

IDL programs and scripts modified and supplied by T. Kelsall extract photometry on a given region at different epochs (see supplements, BATCH.PRO, PSDED.PRO, READ_SAD.PRO, REAQD_SAR.SCR). The following MATHCAD program prepares the input data for a 2D CFFT.

READ_SAD.SCR script gives the selected region information. An area centered on the NEP of size 33 by 33 pixel in the North and East directions is selected for study. In this example three sets of daily photometric values at weekly intervals are extracted for all bands.

The following expression reads a given days photometry within the given region,

$$Data := READPRN(file) \qquad P := rows(Data) \qquad p := 0 .. P - 1$$

The number of total observations is: $P = 1877$

The North - East size of the area is : $ne := 33$

Central Pixel index is: $CPI := 49152$

The waveband is selected by modifying the following expression, (band 5 is selected in this example).

Select photometry band: $band := 5 \quad band := band + 2$

Test sentinel value: $sentinel := -10000$

The following steps are to extract the pixel numbers. First we define the constant indices to be used,

$$N := Data^{<0>} \qquad i := 0..15 \; j := 0..7$$

The total number of pixels on a face by definition is : $nf := 256$

The following function is to determine the face number and then to remove the face pixels,

$$face(x) := x - nf^2 \cdot floor\left(\frac{x}{nf^2}\right)$$

This one is to convert a pixel number to binary,

$$b(x) := floor\left(mod\left(\frac{x}{2^i}, 2\right)\right)$$

The following function is to compute the X and Y components

$$fx(x) := \sum_j x_{2\cdot j}\cdot 2^j \qquad fy(x) := \left[\sum_j \left(x_{2\cdot j+1}\cdot 2^j\right)\right]$$

We now compute the reduced pixel numbers,

$$Nr_p := face\left(N_p\right) \qquad Ncp := face(CPI)$$

and convert them to binaries,

$$bin_{p,i} := b\left(Nr_p\right) \qquad bincp_i := b(Ncp)$$

Using the above function, we compute x and y coordinates as follows,

$$x_p := fx\left[\left(bin^T\right)^{<p>}\right] \qquad y_p := fy\left[\left(bin^T\right)^{<p>}\right]$$

$$xcp := fx((bincp)) \qquad ycp := fy(bincp)$$

We shift the pixel coordinates so that they will act as pointers to the indices of a matrix. Determine the shift from the input ne size,

$$shift := ceil\left(\frac{ne}{2}\right) - 1 \qquad shift = 16$$

Apply shift to x and y indices:

$$x_p := x_p - (xcp - shift) \quad y_p := y_p - (xcp - shift)$$

Put them in a matrix and augment with data matrix,

$$M := \text{augment}(x, y) \qquad M := \text{augment}(M, \text{Data})$$

The following table displays first three rows of the newly formed matrix. Last ten columns shows the photometry at bands 1 through 10.

M =

	0	1	2	3	4	5	6	7	8	9	10
0	0	0	16128	0.21853	0.08033	0.06683	0.61004	14.1721	22.3825	-16400	-16400
1	1	0	16129	0.20412	0.08033	0.06683	0.57273	14.1088	22.7516	-16400	-16400
2	0	1	16130	0.14686	0.05186	0.02423	0.48543	14.0244	22.4235	7.26817	-16400

Form the patch,

$$\text{patch}_{M_{p,0}, M_{p,1}} := M_{p,\text{band}}$$

patch =

	0	1	2	3	4	5	6	7
0	14.1721	13.7606	13.9189	13.7606	14.0455	13.9716	0	0
1	14.1088	13.9716	13.7606	13.8344	0	13.7606	0	0

and set the sentinels to zero,

$$i := 0 .. ne - 1 \qquad j := 0 .. ne - 1$$

$$\text{patch}_{i,j} := \text{if}\left(\text{patch}_{i,j} < \text{sentinel}, 0, \text{patch}_{i,j}\right)$$

We compute the area average (excluding zeros),

$$\text{n\_nonzero} := \sum_j \sum_i \text{if}\left(\text{patch}_{i,j} = 0, 0, 1\right)$$

$$\text{patch\_mean} := \frac{1}{\text{n\_nonzero}} \cdot \left(\sum_i \sum_j \text{patch}_{i,j} \right) \qquad \text{patch\_mean} = 13.34858$$

and replace the zero values with the area mean,

$$\text{patch}_{i,j} := \text{if}\left(\text{patch}_{i,j} = 0, \text{patch\_mean}, \text{patch}_{i,j} \right)$$

Power Spectrum in two dimensions and results

In this section two dimensional complex Fourier transforms (CFFT) are constructed by applying complex fast Fourier transforms in a single dimension to the columns of the input matrix (see *Numerical Recipes* for a discussion about the transformations) and then to the columns of the transposed transform matrix. The resulting complex matrix is then converted to the power spectrum of the patch data.

Define running indices,

$$j := 0 .. \text{ne} - 1 \qquad k := 0 .. \text{ne} - 1$$

and compute the complex fast Fourier transforms of each columns of patch matrix through the following expression,

$$A^{\langle k \rangle} := \text{CFFT}\left(\text{patch}^{\langle k \rangle} \right)$$

We apply the CFFT to the transposed columns of the previous step,

$$T^{\langle k \rangle} := \text{CFFT}\left[\left(A^{T} \right)^{\langle k \rangle} \right] \qquad \text{complex} := T^{T}$$

The power spectrum is obtained through the magnitude of the complex output matrix,

$$\text{PS} := \overrightarrow{\left| \text{complex} \right|}$$

Subsequent Figures show the power spectrum of the selected patch at three daily intervals for band 2, 5, 6, 7, and 10. Three exhibits on each page are the input daily photometry (one week apart), the log of the computed power spectrum shown on a contour map and the 3D surface display of the power spectrum. All bands exhibit the following properties:

1 - Daily minimum and maximum values are in agreement for repeated patches on all bands.

2 - Daily minimum and maximum values of the computed log power spectrum are in agreement on all bands.

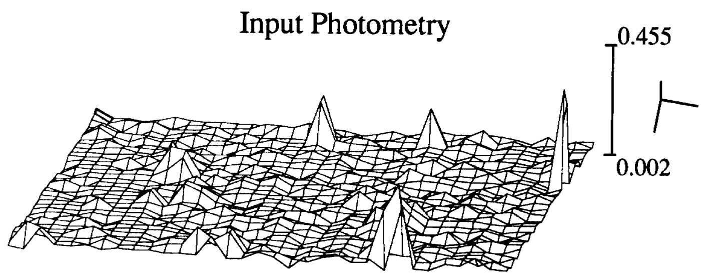
3 - There are no apparent trends or hidden periodicities in the input photometry or in the corresponding power spectrum, except the artifacts created by the missing values.

More insight can be obtained if ,

1 - Larger number of photometric values (density per pixel) such as weekly patches are used in the analysis.

2 - Selection of a more exciting region in the sky will contribute to the richness of the power spectra, and thereby identifiability of variations on constancy.

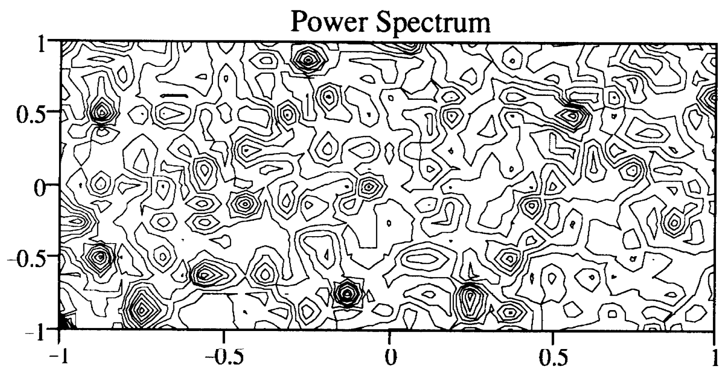
3 - Use of a larger patch with known features can be used to check the sensitivity of the power spectra to disclose such in the frequency domain.



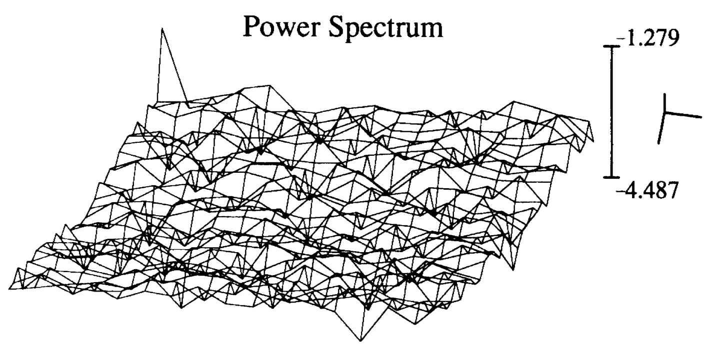
patch

Plot the power spectrum but only the first half

$$i := 0..\frac{ne-1}{2} \quad P^{<i>} := PS^{<i>}$$



$\overrightarrow{\log(P)}$



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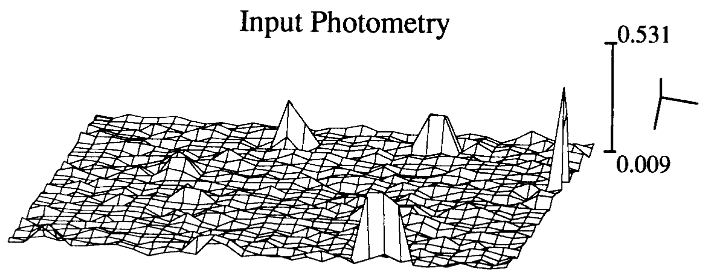

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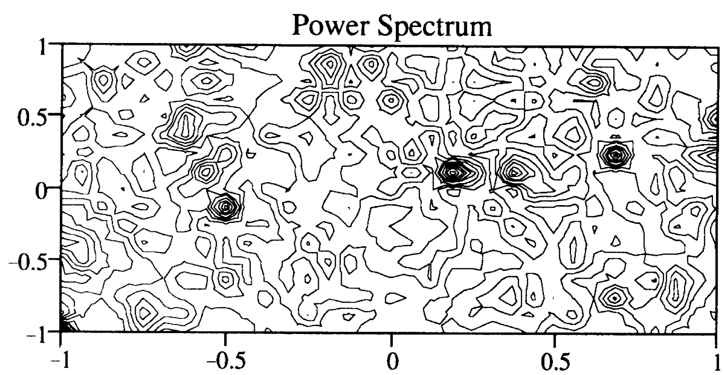

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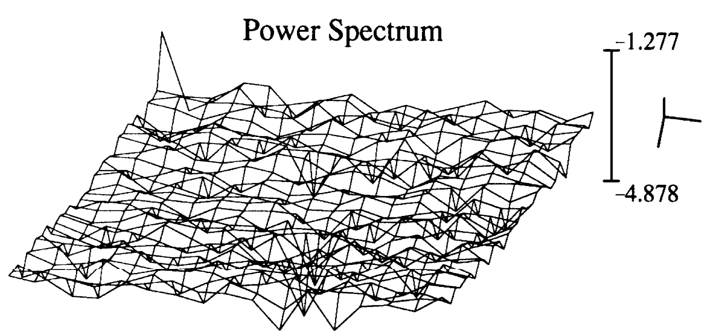

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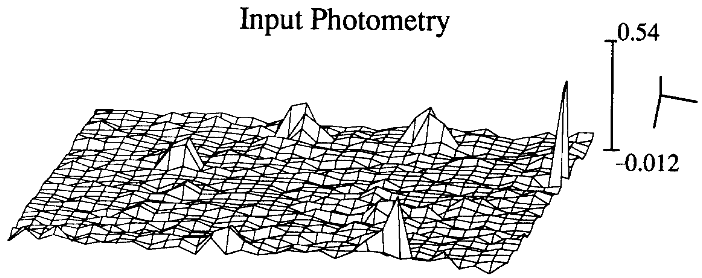

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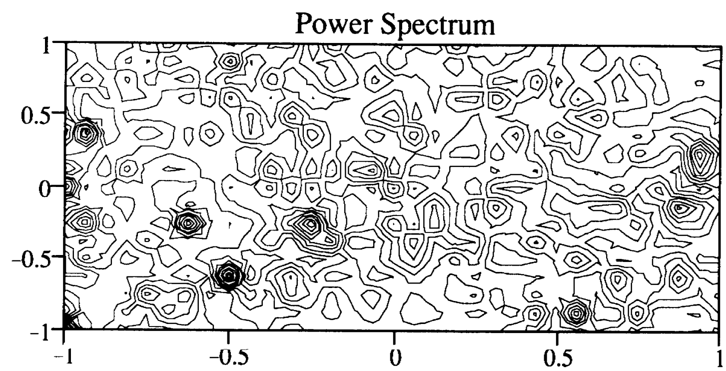

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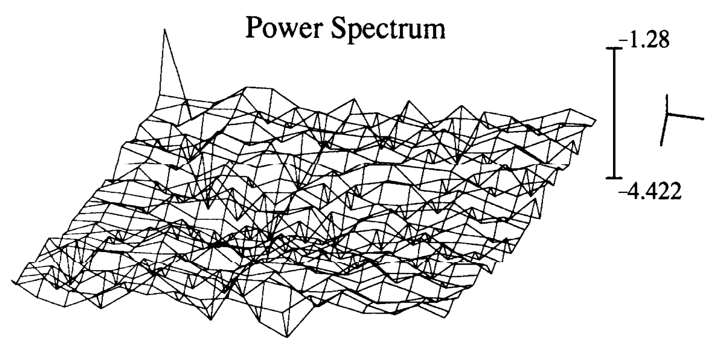

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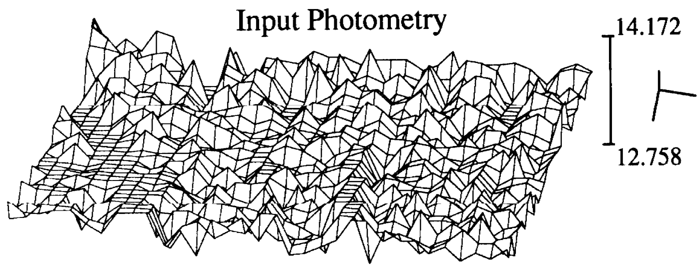

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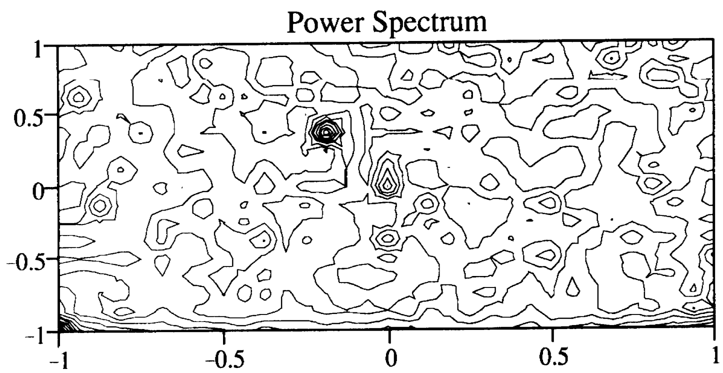

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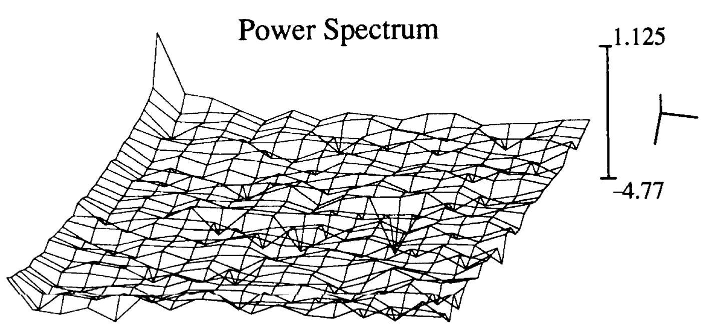
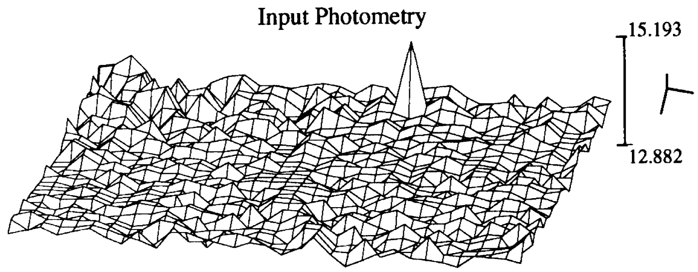


$\overrightarrow{\log(P)}$

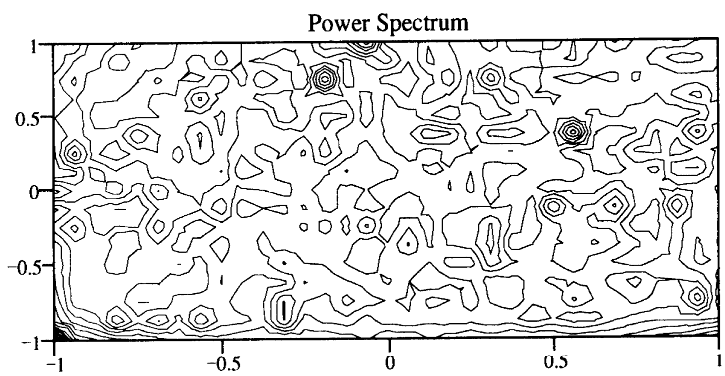
Day 2



patch

Plot power spectrum but only first half

$i := 0 .. \frac{ne - 1}{2}$ $\quad P^{<i>} := PS^{<i>}$



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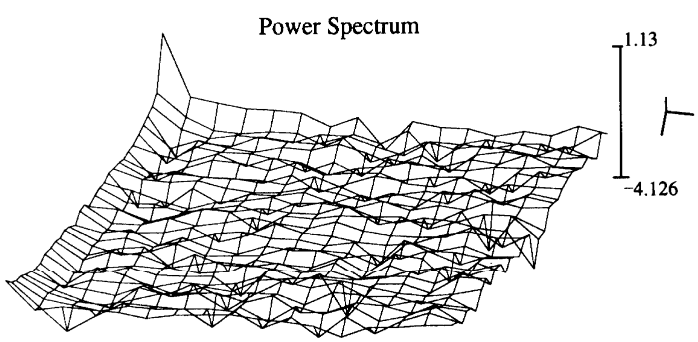

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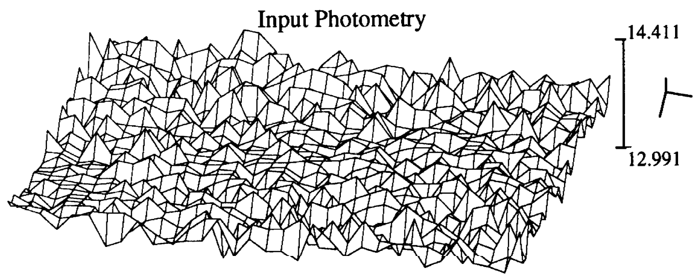
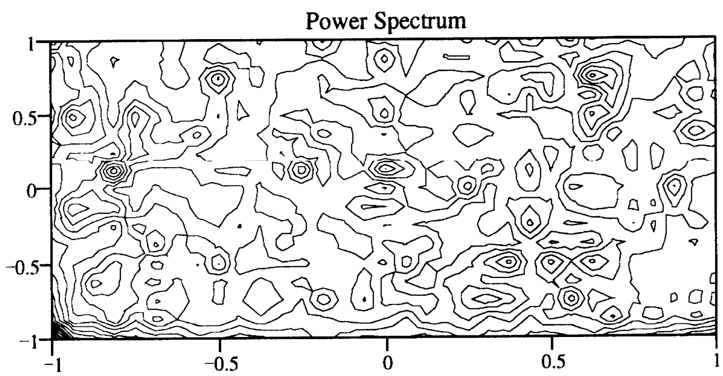
Day 3



patch

Plot power spectrum but only first half

$i := 0 .. \frac{ne - 1}{2} \qquad P^{<i>} := PS^{<i>}$



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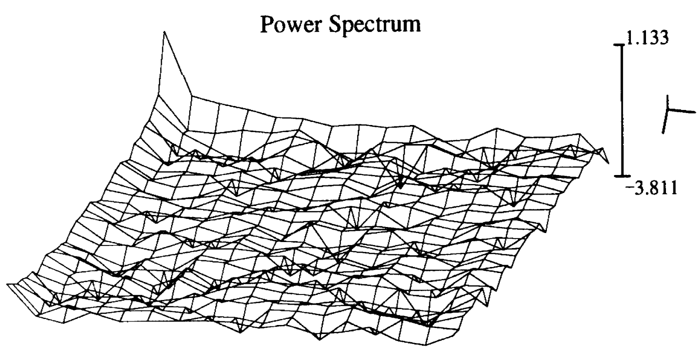

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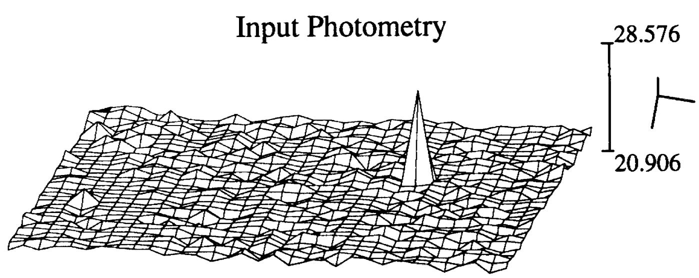

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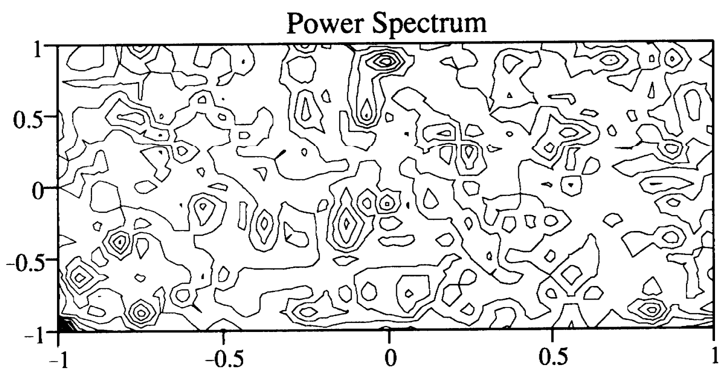

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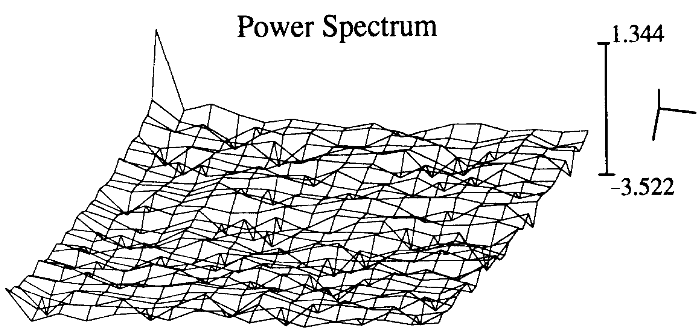

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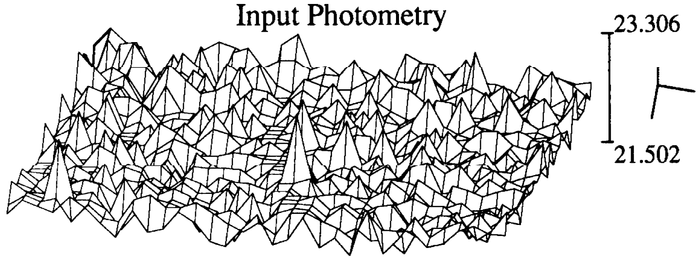
Band 16 day 2

Input Photometry



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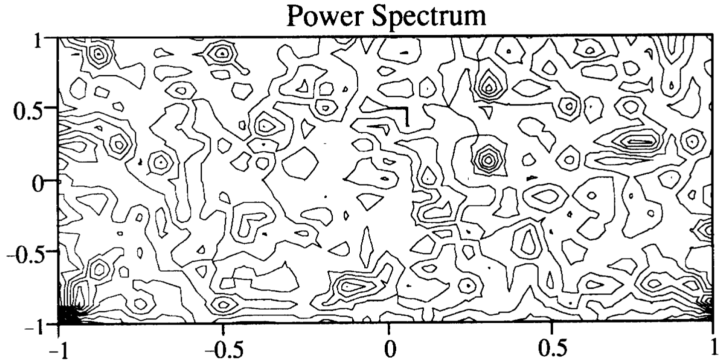
Power Spectrum



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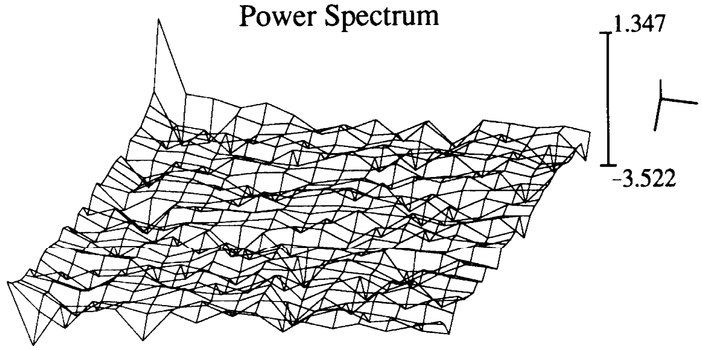
Power Spectrum



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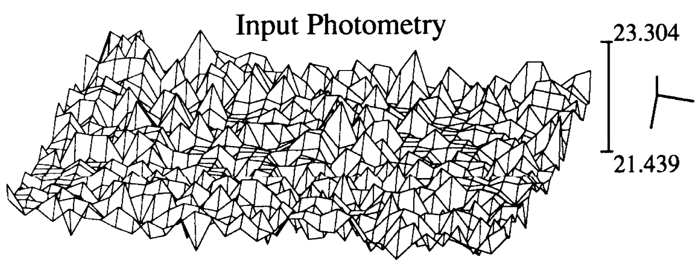

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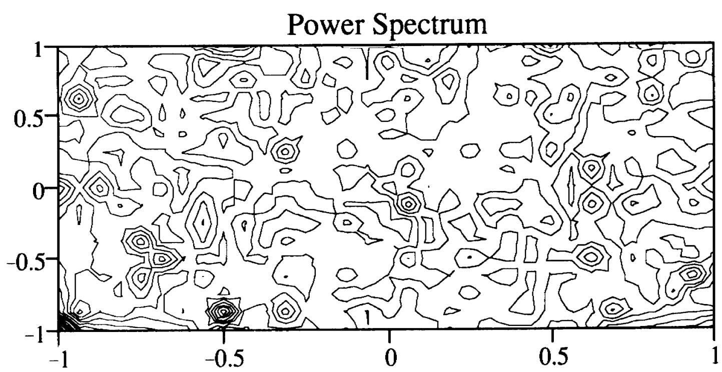

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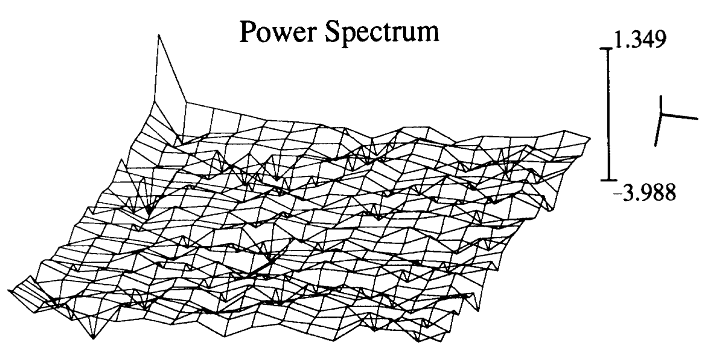

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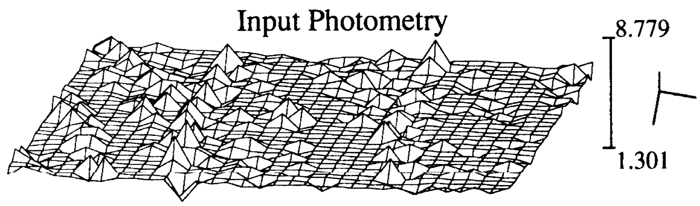

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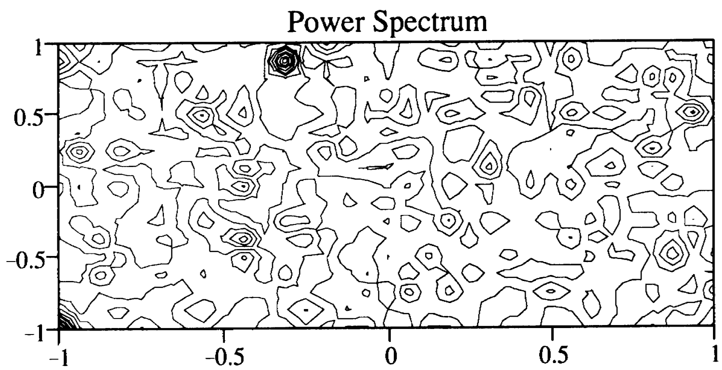
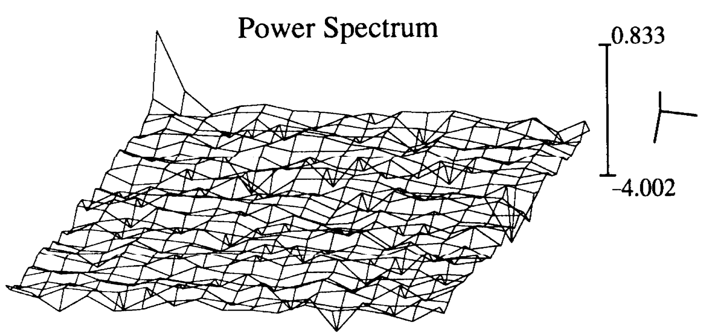


$\overrightarrow{\log(P)}$



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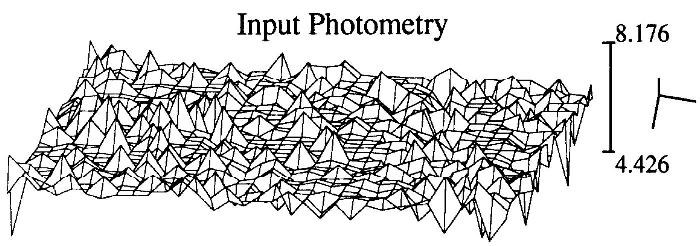

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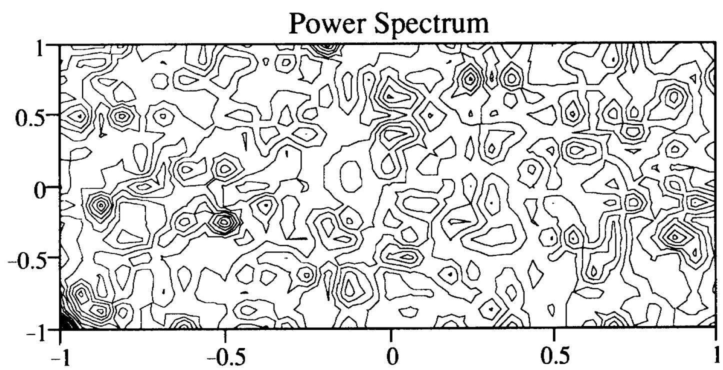

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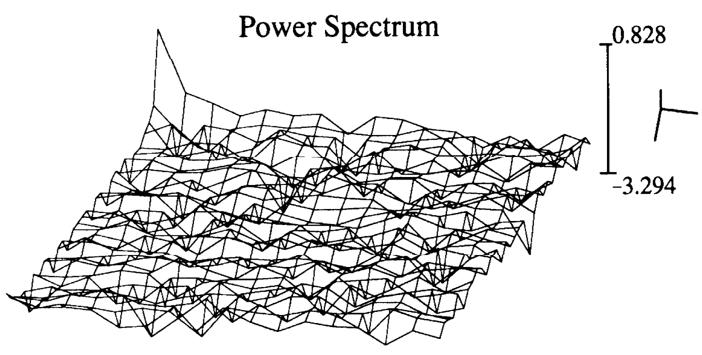

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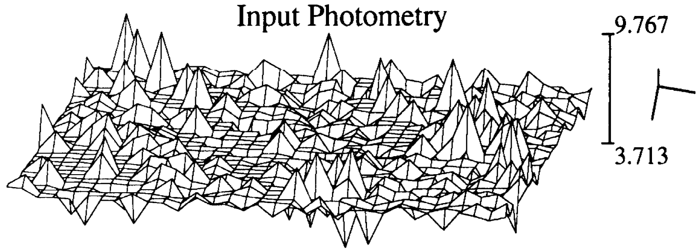

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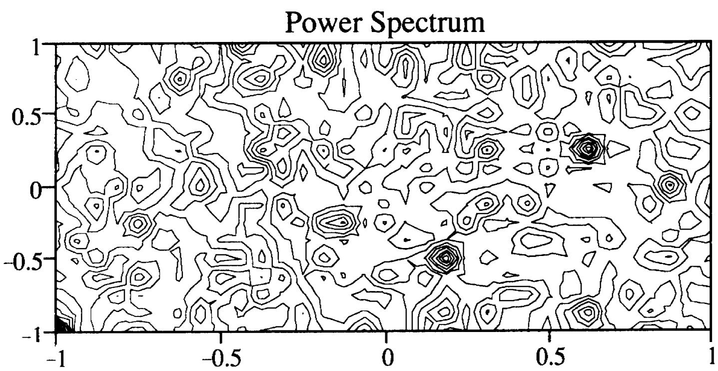
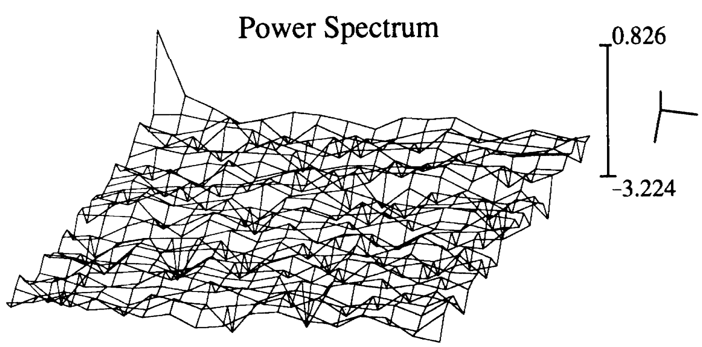
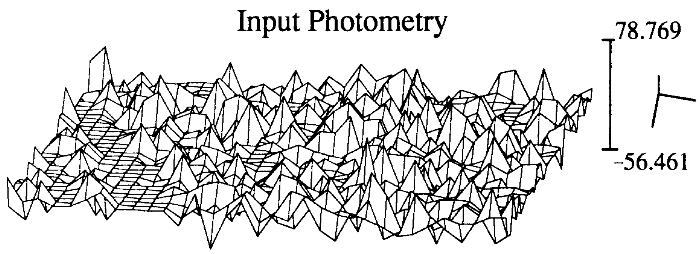


$\overrightarrow{\log(P)}$



$\overrightarrow{\log(P)}$

Band at 40 Day 1



patch

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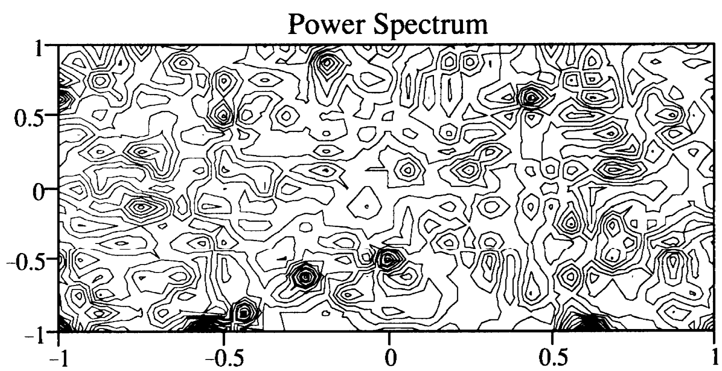
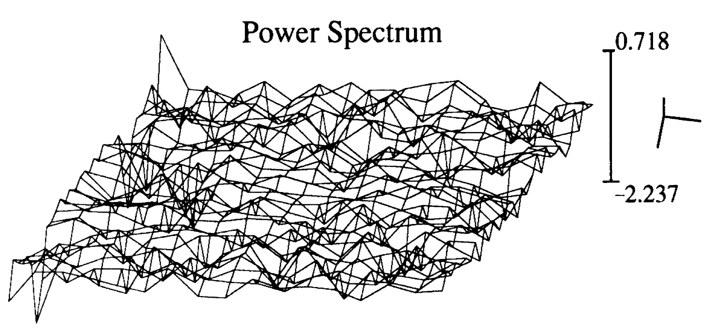
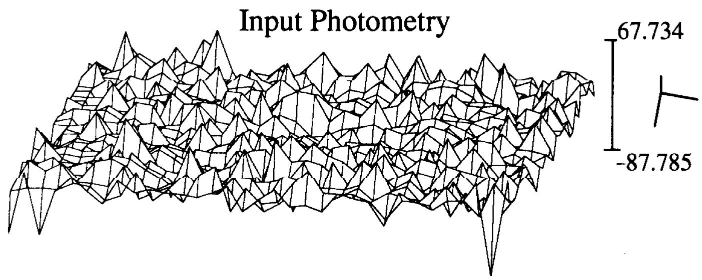


$\overrightarrow{\log(P)}$



$\overrightarrow{\log(P)}$

[illegible]



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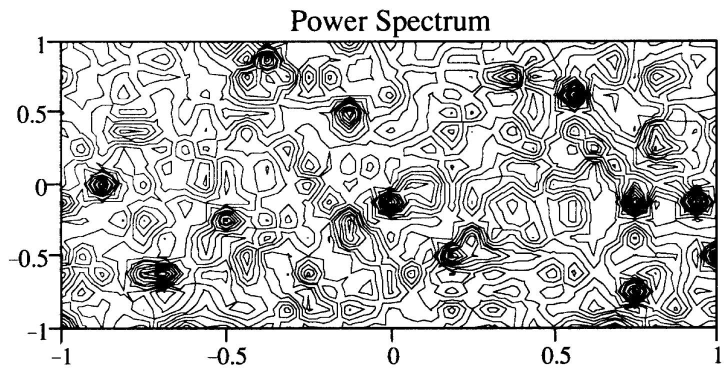

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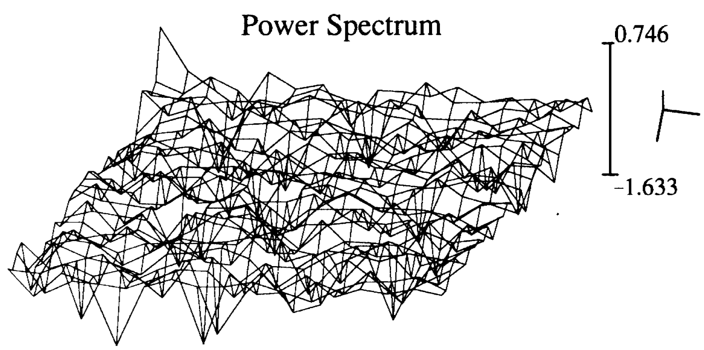

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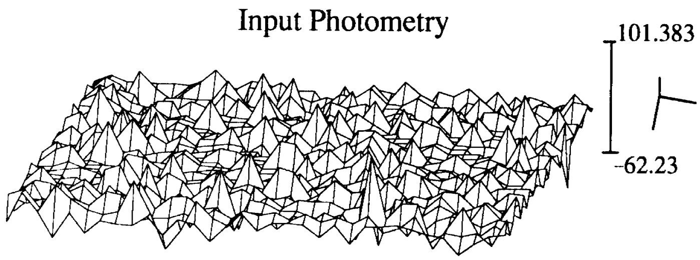

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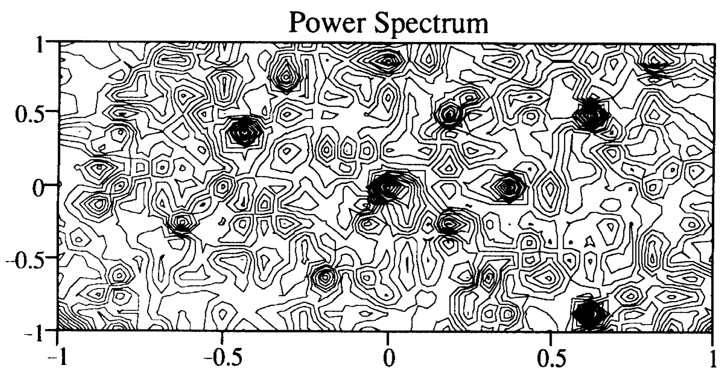

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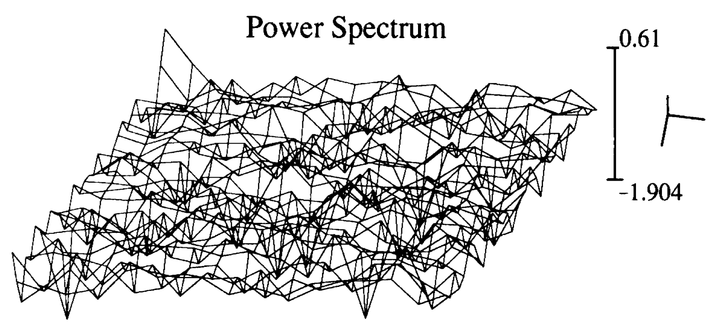

$\overrightarrow{\log(P)}$