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ORIGINAL ARTICLE

Polar motion modeling, analysis, and prediction with time dependent harmonic coefficients

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Abstract A time dependent amplitude model was proposed for the analysis and prediction of polar motion time series. The formulation was implemented to analyze part of the new combined solution, EOP (IERS) C 04, daily polar motion time series of 14 years length using a statistical model with first order autoregressive disturbances. A new solution approach, where the serial correlations of the disturbances are eliminated by sequentially differencing the measurements, was used to estimate the model parameters using weighted least squares. The new model parsimoniously represents the 14-year time series with 0.5 mas rms fit, close to the reported 0.1 mas observed pole position precisions for the x and y components. The model can also predict 6 months into the future with less than 4 mas rms prediction error for both polar motion components, and down to sub mas for one-step ahead prediction as validated using a set of daily time series data that are not used in the estimation.

Keywords Polar motion · Chandler wobble · Prediction · Variable amplitude model · Observation differencing · Autoregressive disturbances

1 Introduction

The study of earth orientation data provides information about the irregular motion of fluids on a global scale, hence serves to the needs of research in geophysics, meteorology, oceanography, geomagnetism, hydrology and other fields (Guinot 1980). Sea level changes, seasonal atmospheric mass redistribution, tidal deformation and angular momentum (winds, ocean currents, motions of the liquid core) all contribute to the pole position and earth rotation (Rochester 1984). Predicted polar motion (PM) information is also needed to improve rapid and long-term satellite orbit predictions, which are of practical importance.

Determination of PM using satellite techniques with a precision of 2 mas was demonstrated as early as 1970s, which was several orders of magnitude better than the earlier astrometric methods (Anderle 1972). Currently, pole positions can be determined with a precision of less than 0.1 mas using space geodesy techniques (Fig. 1).

From daily to sub-decadal time scales, the dynamics of PM is dominated by the Chandler wobble that acts like a resonant oscillator with a nominal period of about 14 months. The forcing function for this dynamic has a wide range of periodic components including secular, interannual, annual and subannual variations of atmospheric, oceanic, and ground water origins. However, despite the significant progress in determining the pole positions by geodetic means, and our understanding of the origin of the excitation of the PM, including the Chandler wobble as ocean bottom pressure (Gross 2000; Brzezinski et al. 2002), and/or tropospheric wind and IB-pressure (Aoyama et al. 2003) being important sources of wobble excitation, empirical modeling and prediction of geodetic PM components remain enigmatic (Chin et al. 2004).

Use of Fourier series is a standard approach for spectral analysis of PM time series followed by a spatial–temporal model, which consists of statistically significant harmonics (hybrid trigonometric time series) and a trend parameter for the secular variations in the pole positions. The resulting trigonometric harmonic series, however, may not always capture

This study is dedicated to the memory of Prof. Urho Uotila (1923–2006) whose teaching of "Adjustment Computations" over the years influenced so much, so many of us who had the privilege of being his students.

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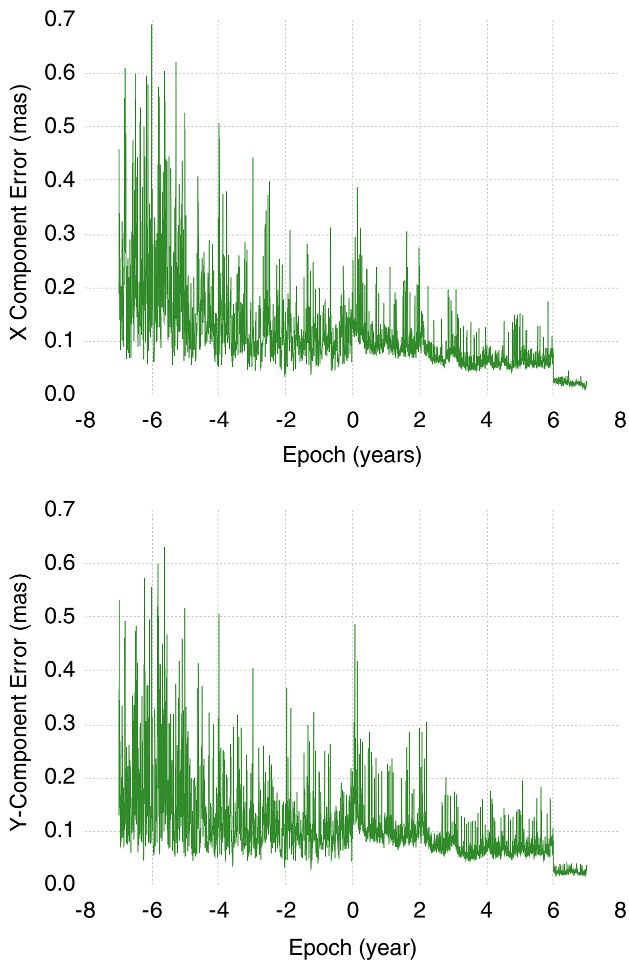

Fig. 1 The pole position errors for the *X* (*above*) and *Y* components. The average error is 0.1 mas for each component. The centered epoch of the series is 31/12/1999

the slow variations in the pole position over time, which are characteristics of the time series (Jeffreys 1940). Hence, extended models are needed for an effective representation for periodicities in the polar motion (Chao 1985).

A number of studies in this area include: Proverbio et al. (1971) who analyzed the Chandler motion with time confirming a correlation between the amplitudes and periods; Nastula et al. (1993) investigated amplitude variations in the Chandler and annual wobbles, including their prediction; Zhu (1981) used an early application of harmonic representation of the polar motion components; Okubo in 1982 discussed Chandler wobble having a single period with a damped oscillator; Chao (1985) expressed each component of PM as a sum of two sinusoids with an annual and Chandler frequencies, respectively, plus an affine function as the trend.

Autoregressive models were also investigated for polar motion prediction as well, as early as 1984 by Chao. Kosek et al. (1998) solved for an optimal estimate based on a criterion to minimize the discrepancy among the sample covariances under various window functions of different lengths. Schuh et al. (2002) deployed a similar model for the PM as Chao (1985) but in addition, they characterized the residual signal with an artificial neural net. Chin et al. (2004) have recently developed models for improved short-term polar motion predictions.

The above brief account of the progress in this area is by no means complete. Detailed discussions about the PM studies can be found in Dick et al. (2000), and Höpfner (2004) who also introduced the use of rotary component analysis (Gonella 1972; İz 2002) into the polar motion analysis.

2 Time dependent amplitude PM model

We start with a new time dependent amplitude model with *fixed* periods (i.e. the wobble periods are assumed to be invariant over time) to represent the PM time series.

Fixed-period model

A fixed-period representation for the PM components can be written as a trigonometric function:

$$y_t = a + b(t - \bar{t}) + \sum_{i=1}^{m} \gamma_i \cos\left(\frac{2\pi}{P_i}(t - \bar{t}) + \phi_i\right) + e_t \qquad t = 1, \ldots, T \tag{1}$$

In this expression, y_t is the observation vector for either X or Y PM components at a given epoch t, and $\bar{t}$ refers to the middle epoch of the series. The parameter a is the intercept of a trend with slope b that represent the secular variations in the PM components to be estimated. γ_i is the amplitude of the corresponding ith modeled wobble with a *known period P (fixed)*, and *unknown* phase angle ϕ_i.

For m modeled wobbles with periods P_i of a time series of length T, this model is non-linear in terms of the unknown amplitude and phase parameters, and can be replaced by the following linear representation:

$$y_t = a + b(t - \bar{t}) + \sum_{i=1}^{m} \alpha_i \sin\left(\frac{2\pi}{P_i}(t - \bar{t})\right) + \sum_{i=1}^{m} \beta_i \cos\left(\frac{2\pi}{P_i}(t - \bar{t})\right) + e_t \tag{2}$$

where, α_i, β_i are the harmonic coefficients (*Fourier*) that are related to the unknown wobble parameters by

$$\gamma_i = \sqrt{\alpha_i^2 + \beta_i^2} \qquad \phi_i = \arctan\left(\frac{\alpha_i}{\beta_i}\right) \tag{3}$$

The disturbances, e_t, (measurements errors) are *assumed* to be uncorrelated, stationary, and either homogeneous or heterogeneous for both representations. This model, which has been extensively used in the past (Zhu 1981; Schuh et al. 2002), assumes that the periods of the wobbles are known accurately. Yet the spectrum of the wobbles, especially Chandler wobble, cannot be accurately represented by a known single frequency, because of the presence of broadband nearby frequencies (Pan 2007; Vicente and Wilson 2002). A variant of this model is proposed in the following section.

Polar motion model with time dependent harmonic coefficients

Modeling the Chandler wobble amplitude as a function of time was first investigated by Chao (1985) using an exponential model, but reported to be unsuccessful in the prediction of polar motion components. The proposed variable amplitude model is a simpler one, which is stated as follows:

$$y_t = a + b(t-\bar{t}) + \sum_{i=1}^{m} \alpha(t)_i \sin\left(\frac{2\pi}{P_i}(t-\bar{t})\right) + \sum_{i=1}^{m} \beta(t)_i \cos\left(\frac{2\pi}{P_i}(t-\bar{t})\right) + e_t \quad (4)$$

This formulation differs from the fixed-period model given by (2), in their harmonic coefficients of the trigonometric series. The new formulation allows the harmonic coefficients to vary *linearly* in time. The linear rates of changes in the harmonic coefficients, denoted by $\dot{\alpha}_i$ and $\dot{\beta}_i$, are introduced together with their corresponding intercepts as additional unknown parameters to be estimated as follows:

$$\alpha(t)_i := \alpha_i^0 + \dot{\alpha}_i(t-\bar{t}) \quad (5)$$

$$\beta(t)_i := \beta_i^0 + \dot{\beta}_i(t-\bar{t}) \quad (6)$$

In these expressions, α_i^0 and β_i^0 are described as the *nominal* values of the harmonic coefficients (intercepts of the trends). They are related to the variations in the wobble amplitudes $\gamma(t)_i$ and phases through the following expressions:

$$\gamma(t)_i = \sqrt{\alpha(t)_i^2 + \beta(t)_i^2}, \quad \phi(t)_i = \arctan\left(\frac{\alpha(t)_i}{\beta(t)_i}\right) \quad (7)$$

The new parameterization modulates the fixed-period sin and cosine functions as shown in Fig. 2, which are produced by evaluating a synthetic wobble with a period of 430 days, and harmonic coefficients with magnitudes of 100 mas. The rates of changes in the harmonic coefficients are 0.001 mas/year for a hypothetical 14-year series. The linear rates produce a time dependent amplification in the harmonic coefficients similar to the variations as observed in the original PM data for 6–7 years.

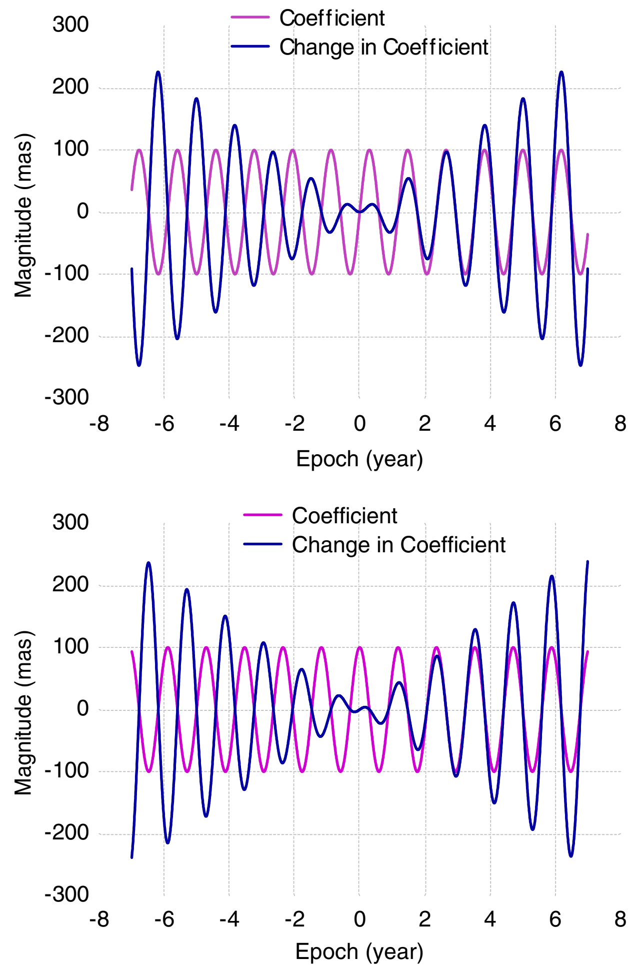

Fig. 2 The contribution of a synthetic wobble with a period of 430 days is displayed for a series of 14 years. The harmonic coefficients' magnitudes are 100 mas and their rates of changes are 0.001 mas/year. Sine (*above*) and cosine terms from (4) are shown

Statistical model

The earlier studies assumed that the measurements are independent (Zhu 1981; Schuh et al. 2002; Höpfner 2004), i.e. the series are not serially correlated. Although a number of earlier (Chao 1972) and recent studies (Chin et al. 2004) have used autoregressive models for modeling and for the prediction of PM components, these studies targeted modeling the exciting functions themselves (not the disturbances) as the source of autoregressive processes, which may be valid for modeling empirical variations in the series regardless of their origin (like the prevalent use of polynomials for similar purposes).

Meanwhile, İz and Schaffrin (2001) have shown, using a variate-differencing algorithm, that the early astrometric PM measurement' uncertainties agree well with the ones estimated independently from the variations in the data. Their results provided evidence that with increasing precision and frequency of recent measurements from satellites, the stochastic component of recent PM measurements are serially correlated (İz and Schaffrin 2001). Smoothing and interpolation of PM components induce further autocorrelations in the series. Hence, the statistical model for the *model disturbances* must take into account an autoregressive process, which is represented as a first order process in this study as follows:

$$e_t = \rho\, e_{t-1} + v_t \tag{8}$$

where ρ is the first order *unknown* autocorrelation coefficient, $\{v_t\}$ is the stochastic process with the following *assumed* properties:

$$\begin{aligned} &E(v_t) = 0,\ \ E(v_t^2) =: \sigma_t^2,\ \ E(v_t v_{t'}) =: 0,\ \ \text{for } t \neq t' \\ &\ \Rightarrow E(e_t) = 0,\ \ E(e_t^2) = \sigma_v^2(1-\rho^2) =: \sigma^2 \end{aligned} \tag{9}$$

It can be shown that the corresponding variance covariance matrix for the model disturbances can now be expressed as (İz and Chen 1999)

$$\Sigma = \sigma^2 \begin{bmatrix} 1 & \rho & \rho^2 & \dots & \rho^{T-1} \\ \rho & 1 & \rho & \dots & \rho^{T-2} \\ \vdots & \vdots & 1 & \dots & \vdots \\ \rho^{T-1} & \rho^{T-2} & \rho^{T-3} & \dots & 1 \end{bmatrix} \tag{10}$$

The solution for the time variable harmonic coefficients with its statistical model can be obtained using a two stage approach where, first, the correlation coefficient is estimated from the residuals of an approximate solution, which is then used in the above variance covariance matrix solution (İz and Chen 1999).

A significant assumption in this approach is that the stochastic process is stationary and the corresponding disturbances are homogeneous. Yet, it is evident from Fig. 1 that this assumption is not valid. Although the above statistical model accounts for the effect of serial correlation (autocorrelation), it fails to properly model the weights. The impact of improper modeling of the disturbances may degrade the model solution and the predictions especially in the presence of unmodeled systematic effects.

Another deficiency of the early and the current spatial-temporal formulation of the time series is the difficulty of proper scaling of the resulting normal matrix of the observation equations because of the high correlation among some of the model parameters with similar periods, as it will be discussed in the following section.

Model transformation via differencing

Varying weights are an indication of departures of measurements caused by a number of unmodeled effects that may be present in less accurate measurements, which will bias the trend estimates. Particularly, when significant variations in weights occur at the beginning or at the end of the series, and if these are not properly accounted for in the solutions, the other estimates would be significantly affected (İz and Shum 2000; İz 2006).

Moreover, unmodeled long periodic and secular polar motion variations, because of the finite length of the series, influence the estimates (spectral smearing, Pan 2007) hence, adversely affect the estimation of the model parameters and consequently, prevent the successful prediction of the long-term pole positions. The following transformation ameliorates a number of related problems including the introduction of the weights into the previously discussed statistical model solution.

The model given by (4) can be rewritten as

$$y_t = x_0 + a_{1t}x_1 + a_{21t}x_2 + \cdots + a_{1t}x_{Kt} + e_t \tag{11}$$

in which, x denotes the parameters to be estimated and a are the corresponding known coefficients. If this expression is evaluated at the preceding epoch, $t-1$, and multiplied by ρ and subtracted from itself which is evaluated at t as shown in (11), then

$$\begin{aligned} \Delta y_t &= (1-\rho)x_0 + \Delta a_{1t}x_1 + \Delta a_{21t}x_2 \\ &\quad + \cdots + \Delta a_{1t}x_{Kt} + v_t \end{aligned} \tag{12}$$

where

$$\begin{aligned} \Delta y_t &:= y_t - \rho y_{t-1} \\ \Delta a_t &:= a_t - \rho a_{t-1} \\ v_t &:= e_t - \rho e_{t-1} \end{aligned} \tag{13}$$

Observe that the differencing leave the unknown parameters x_K invariant, therefore, the transformed observation Eq. (12) can be solved using a LS solution with a diagonal v/c matrix of the stochastic process $\{v_t\}$ which is given by

$$\Sigma = \sigma^2 \begin{bmatrix} v_1 & 0 & 0 & \dots & 0 \\ 0 & v_2 & 0 & \dots & 0 \\ \vdots & \vdots & & \dots & \vdots \\ 0 & 0 & 0 & \dots & v_T \end{bmatrix}. \tag{14}$$

If the correction coefficient is *high* (near to one), then the following approximations can be made:

$$\begin{aligned} \Delta y_t &:= y_t - \rho\, y_{t-1} \cong y_t - y_{t-1} \\ \Delta a_t &:= a_t - \rho\, a_{t-1} \cong a_t - a_{t-1} \\ v_t &:= e_t - \rho\, e_{t-1} \cong e_t - e_{t-1} \end{aligned} \tag{15}$$

and (12) reduces to

$$\Delta y_t = \Delta a_{1t}x_1 + \Delta a_{21t}x_2 + \cdots + \Delta a_{1t}x_{Kt} + v_t \tag{16}$$

This approximation is reasonable especially for the *daily* PM time series, which are highly correlated because of the frequency of the solutions, solution techniques (combination of various space geodetic methodologies), smoothing methods, and due to the presence of slowly varying long periodic variations and secular trend. A number of interesting properties of this model are as follows:

1. In this expression, the intercept (the first term) disappears.
2. The coefficient of the slope term is equal to *one* if daily time series are used.
3. The independent stochastic process $\{v_t\}$ is now isolated from the autoregressive contribution; the observation equation is now free from serial correlation of the disturbances. Hence, the transformed observation equations can be solved using the weighted least squares approach with a diagonal variance covariance matrix with heterogeneous variances.
4. If all the systematic variations in the series are properly accounted for, then the residuals reflect the accuracy of the measurements, i.e., the rms solution residuals should be close to the reported sub-mas measurement precision of modern satellite born techniques. In addition, in the presence of high first order autoregressive disturbances, any solution that does not introduce the correlation of the observations in the solution cannot properly reduce the residuals close to their expected precisions.
5. The regular (undifferenced) PM measurements exhibit large power in their frequency spectrum because of the long periodic variations, which is the motivation for the use of high pass filters in detecting significant frequencies in the measurements. The frequencies of the differenced measurements are distinct and extremely energetic because the differencing eliminates slowly varying changes effectively from the measurements, especially for the daily PM time series. In other words, differencing acts like a high pass filter leaving only a number of frequencies that are important for model building and prediction.
6. The geometric properties of the variance-covariance matrix of the parameters for the differenced observation equations, as it will be demonstrated in the following sections, are markedly different from the variance-covariance matrix of the estimates from the regular observation equations.

In the top of Fig. 3, synthetic PM components were produced using the model (4) by simulating a wobble for a number of periods, and harmonic coefficients with magnitudes 100 mas. The rates of changes in the harmonic coefficients are 0.001 mas/year for a series of 14 years. The bottom figure was produced using (12) with the same parameters. Observe that, although the wobble parameters are the same, their signatures on the observations are markedly different.

Similarly, in Fig. 4, the corresponding a priori standard deviations of the estimates for a number of wobble parameters listed in Table 1 are displayed. The top figure shows that

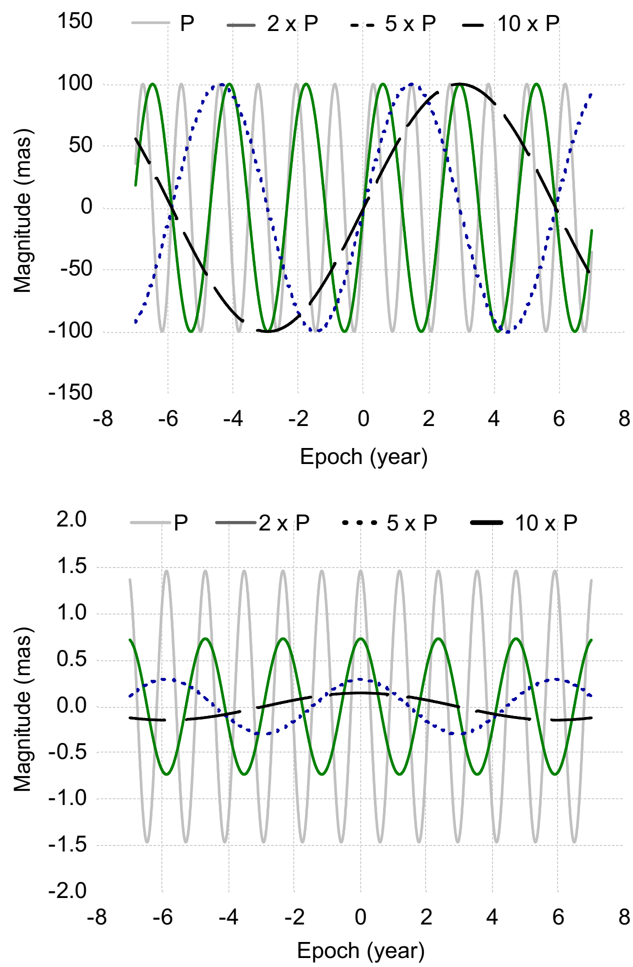

Fig. 3 In the *top figure*, synthetic PM components are produced ,for a series of 14 years, using the model (4) by simulating a wobble for a number of periods whose harmonic coefficients are 100 mas with 0.001 mas/year rates of changes. The *bottom figure* is generated using model (12) using the same parameters

the standard deviations are more or less uniform for all the harmonic coefficients and their rates, whereas the standard deviations of same parameters for the long periodic wobbles for the differenced observation equations increases exponentially.

This, otherwise undesirable property of the differenced model of observation equations, is an opportunity for the inversion problem in this study to more or less *unbiasedly* estimate a number of harmonic coefficients without modeling long periodic wobbles in the solutions including trend, and hence, without getting the estimates unduly inflated by high correlation. Once the shorter harmonic coefficients of the wobbles are estimated, the remaining signatures of the unmodeled wobbles will appear in the residuals as lump sum, quasi-constant effects, which can be removed and used successfully in the prediction process.

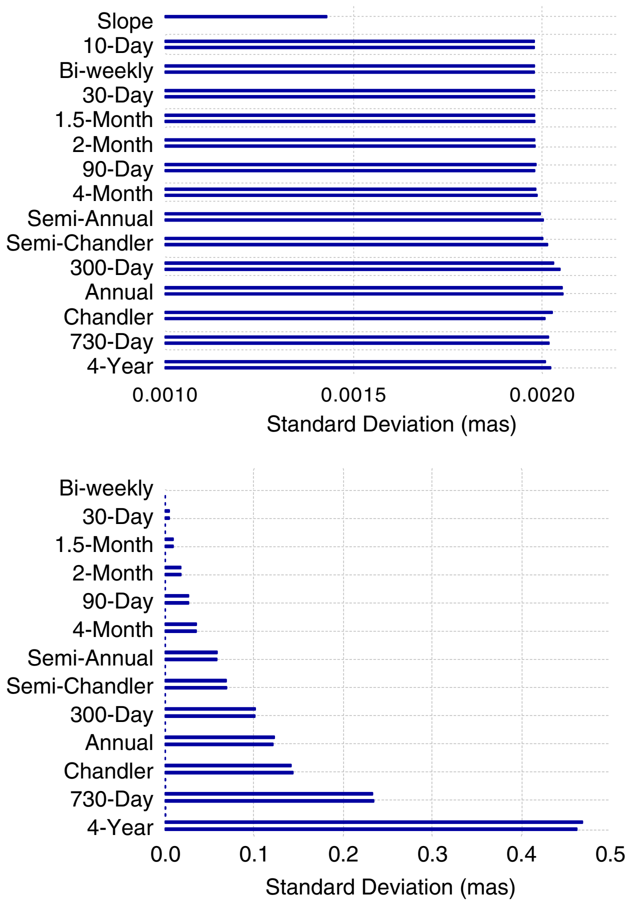

Fig. 4 The a priori standard deviations of the model parameters to be estimated calculated for a number of wobble parameters listed in Table 1 using model (4), regular observation equations, and model (12), the transformed (differenced) observation equations. *Double bars* are for the standard deviations (mas) for the sin and cosine coefficients of the nominal harmonics (*lower and upper part*, respectively). The coefficients for the rate of changes (located in between the *double bars*) are too small to display (mas/day) but exhibit the same patterns

3 PM time series and the initial wobble periods

The Earth Orientation Center of the IERS has recently announced a provisional international reference time series for the Earth Orientation Parameters (EOP), namely C04 (Combined 04) referenced to the ITRF 2005, IGN solution. The new time series is a combination of a number of operational EOP series from a number of analysis centers, which produced their own series using different space born geodetic measurement techniques. The new solution is an improvement over the old official solution, and it is updated two times per week. Further information about the new solution algorithm can be found in Bizouard and Gambis (2007).

This study involves two different data sets that were extracted from the operational C04 series. The first series, 01/01/1993 – 12/31/2006 (Fig. 5), span 14 years, which is approximately twice the length of the Chandler and annual wobbles beat period. The second data set, 01/01/2007 – 06/29/2007, spanning approximately 6 months, was not included in the solutions in order to use it as a control to validate the new model predictions.

The average error, 0.1 mas and less for x and y polar motion components of the reported daily standard deviations, reflects well the accuracy of the state-of-the-art solutions (Fig. 1) as it will be confirmed as a byproduct of this study. The standard deviations of the daily pole position components are, however, not uniform. Both x and y series' standard deviations were higher during their initial years. Therefore, weighted least square solutions are needed for the analysis of heteroscadastic data sets (see İz and Shum 2000; İz 2006 for the effect of systematic effects at the end and the beginning of the series in the modeling mean sea level changes for the tide gauge measurements, which is also valid for this problem).

The solutions require an initial set of periods for the presumably known wobbles, some of which will be adjusted (tailored) during the course of the analysis. These periods arise from a number of frequency bands; from the ocean excitation of polar motion including intraseasonal with periods from several days to 1 year; seasonal variations, Interannual with periods from 1 to 6 years; and decadal changes with periods longer than 6 years including a secular trend for the polar wander.

Table 1, self-explanatory in content, was produced from Höpfner (2004). It contains the initially adopted periods in this study in detail. In the following solutions, these periods were modified iteratively from a number of solutions until their signatures from the time series residuals were reduced or disappeared. In addition, a number of real or pseudo intraseasonal variations as a result of spectral smearing of unmodeled long periodic wobbles that are not listed in the in the table were detected one by one from the spectral analysis of each solution's residuals using Burg's algorithm, (Burg 1975), and incorporated subsequently into the models.

4 Solution and analysis

First, a preliminary solution based on the transformed (differenced) observation equation model was carried out using the periods reported in Table 1 for the 14-year differenced series data (Fig. 6). The residuals' spectra from this solution were then examined for leftover frequencies. The newly detected periods were used in the subsequent solutions and checked whether they decrease the rms residuals. This process continued until all the frequencies that contributed to the residuals were exhausted.

Meanwhile, if some of the new wobble periods turned out to be close to their a priori wobble values, then the a priori wobble periods were replaced with the new periods and additional solutions were obtained until the solution residuals

Table 1 Components of polar motion as reported by Höpfner (2006)

Components of polar motion Phenomenon	Period	Amplitude	Causes
Low-frequency part			
Secular polar motion (True polar wander)	–	ca. 0.4″/century towards 75° W	Post-glacial rebound, melting of glaciers, changes in ground water storage, etc.
Long-periodic variations (Decadal variations)	9, 14, 26, 76 years	ca. 0.02″	Global mass retribution, core-mantle coupling, inner-core rotation
Markowitz wobble	ca. 30 years	ca. 0.02″	
Dominant components			
Chandler wobble (Free motion)	410–442 days	0.03 to 0.27″	Ocean-bottom pressure changes, and fluctuations in atmospheric pressure
Annual wobble (Forced motion)	356–376 days	0.05 to 0.16″	Seasonal air and water mass redistributions
Other periodic oscillations with smaller amplitudes			
Quasi-biennial wobble	About 2 years	up to 8 mas	dynamics of the atmosphere and the oceans
300-Day wobble	290–300 days	up to 4 mas	
High-frequency part			
Semi-Chandler wobble	200–240 days	up to 8 mas	Lateral heterogeneities, bifurcation solution
Semiannual wobble	170–200 days	up to 120 mas	As for the annual wobble
	4 months	up to 6 mas	Dynamics of the atmosphere and the oceans
	90 days	up to 3 mas	
	2 months	up to 5 mas	
	1.5 months	up to 4 mas	
	1 month	ca. 1 mas	
	0.5 months		
	10 days	ca. 1 mas	
Diurnal motion			Ocean tides
Sub-diurnal motion			

remained insensitive to the newly added or modified wobble periods.

Figure 7 displays the histograms of the final residuals for the differenced observation equations for both components. Significant portion of the residuals fall within 0.5 mas interval and they are normally distributed. The rms residuals for the x and y polar components are 0.6 and 0.4 mas respectively. These statistics are close to the 0.1 mean measurement errors.

Table 2 lists the amplitudes derived from the estimated harmonic coefficients with rates using the full variance–covariance matrix of the estimates. All the other parameters have periods less than the Chandler wobble period, except one interannual wobble with a period of 512 days that was inferred from the spectrum of the residuals. Overall, the wobble periods did not change much except two new wobbles were detected and deployed with periods 313 and 258 days. Once these periods were introduced, the removal of the semi-chandler period did not alter the residuals. Therefore, the semi-Chandler period was removed from the final solution for a parsimonious representation of the variations. The new wobbles are more likely to be seasonal as their presence was detected by Schuh et al. (2001) using wavelet analysis.

Note that the estimation of these closely spaced period coefficients is rather difficult using regular trigonometric models because of their high correlations. The new formulation, however, is robust to the changes in the initial nominal periods, which are promptly compensated (balanced) by the changes in their amplitude rates. The rms residuals remained practically invariant when the nominal periods were modified.

Considering the magnitudes and estimated uncertainties of the wobbles, additional wobbles parameters can be removed from the model to keep the residual serial correlation at a minimum. Among them, the wobbles whose periods are less then four months do not exhibit any variations in their amplitudes, most likely because of their well defined tidal origins.

The first order correlation coefficient was also calculated for each solution residuals. The estimated coefficient value decreased slowly with the inclusion of significant wobble parameters but leveled around 0.8 for the final solution.

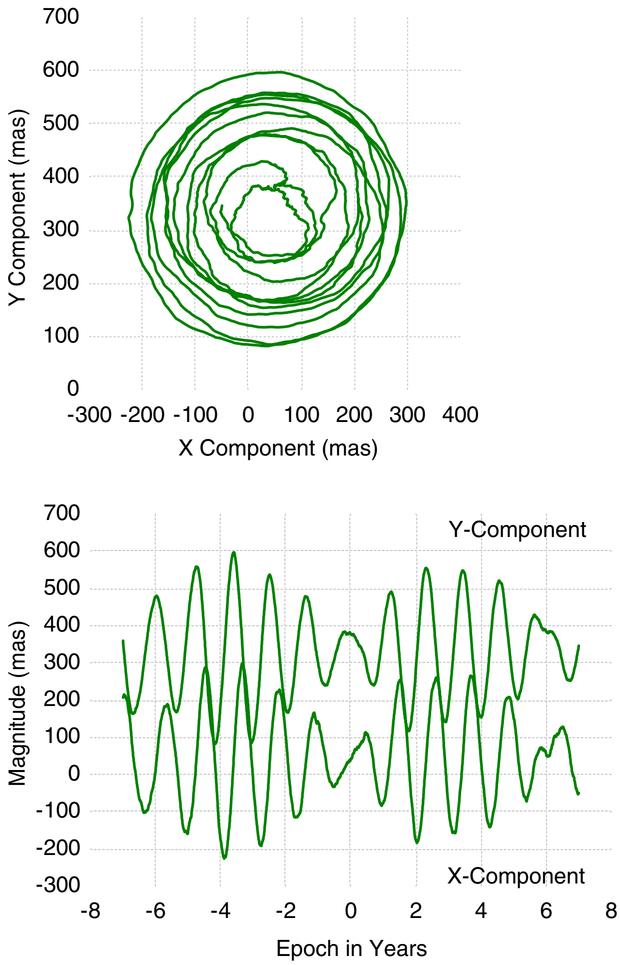

Fig. 5 The daily-PM time series, IERS C04, used in this study for the *x* and *y* components. The series span through 01/01/1993 – 12/31/2006. The centered epoch of the series is 31/12/1999

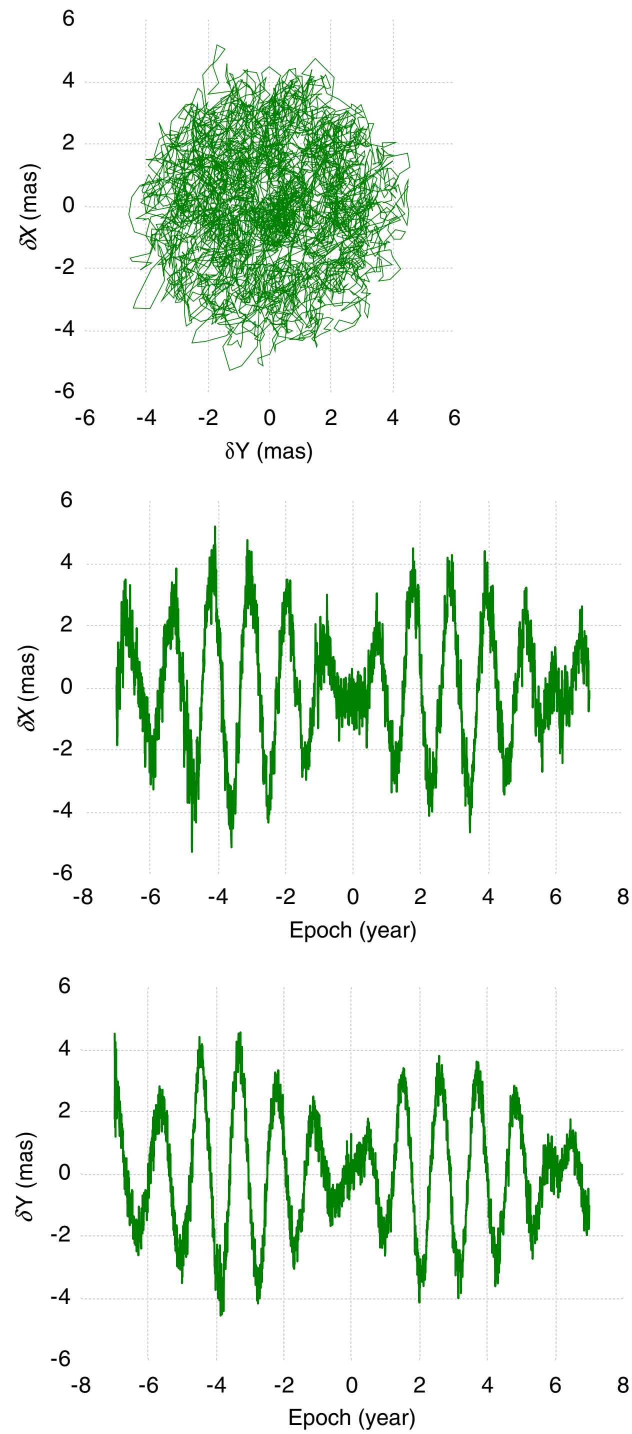

Fig. 6 The *top figure* shows the plot of the sequentially differenced PM series components (*bottom plots*) against each other. The differenced series effectively cancels long periodic effects and reveals the energetic behavior of the series

The estimated harmonic coefficients of the differenced observation equations were used to construct the original model (4) and to calculate the residuals for the original PM observations. Figure 8 displays the model, original observations, and the residuals for both PM components. The residuals are biased because the intercept parameters were eliminated by differencing the differenced observation equations, and the long term and secular wobbles were not included in the model. Although the sum of the residuals of the differenced observations equations is equal to zero, the residuals of the original observation equations are not because they were indirectly derived. The systematic offsets in between the model and the observations that are displayed in Fig. 8 are the lump-sum effects of these unmodeled variations. The residuals for both components were corrected by their mean values to center them about zero. The rms residuals for the mean corrected residuals are 11.2 and 12.3 mas for the *x* and *y* components respectively. As expected, they are much larger than the residuals of the differenced observation equations, which are functions of the independent disturbances only. It is important to note that all the residual statistics reported so far can further be reduced by a factor of square root of five, if 5-day averaged PM data are used instead of daily series.

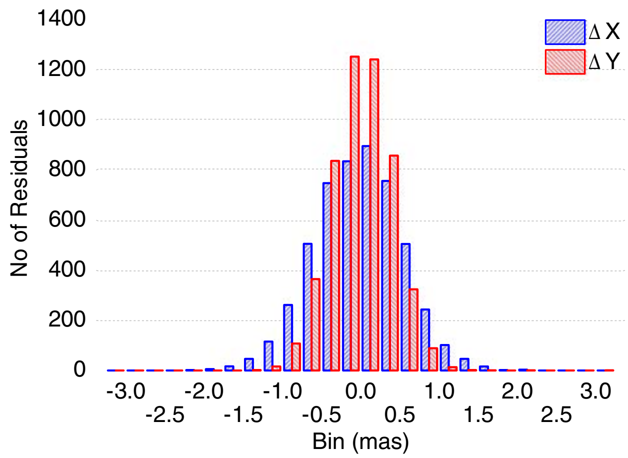

Fig. 7 Histogram of the residuals calculated from the differenced model for the x and y components

The trend parameters calculated from the residuals are approximately 0.2 mas/year for both components for the series, which is significantly less than the expected 3.3 mas/year as reported by Schuh et al. 2001 that were calculated using much longer time series including early astrometric observations.

5 Prediction

A best fitting model to represent the PM data can be useful as a filter for a number of investigations, such as to investigate the impact of the earthquakes on the pole positions, which was the original motivation of this study. Yet, its usefulness is limited by the degree the model can predict, regardless of how parsimonious and representative it is, of the data.

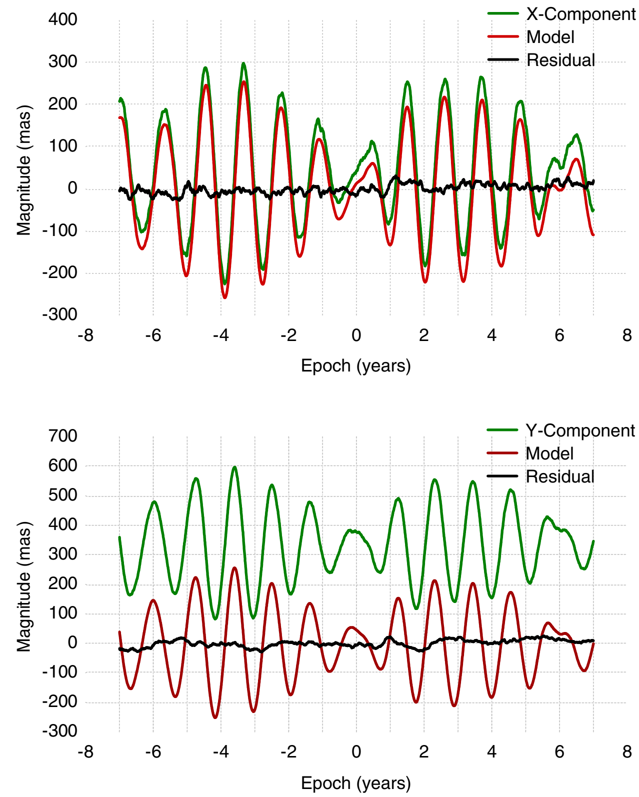

Fig. 8 The *top figure* shows the original PM data for the x-component (y-component for the *bottom figure*), the PM values generated using the model with harmonic parameters estimated from the differenced observation equations. The offsets between the observed and model derived values are because of the omission of the long periodic effects including the trend parameters. The residuals were corrected with their average values. The rms residuals are 11.2 and 12.3 mas for the x and y components, respectively

Table 2 Estimated amplitudes and their rates for the PM wobbles whose periods were inferred from the residuals of the solution using the differencing model

Wobble type	Period (day)	Amplitude x (mas)	Amplitude rate x (mas/year)	Amplitude y (mas)	Amplitude rate y (mas/year)
Interannual	512.0	12.5(1.8)	−0.9(0.002)	13.0(1.7)	−1.8(0.002)
Chandler	429.5	148.0(1.5)	−6.0(0.001)	147.4(1.5)	−5.7(0.001)
Annual	365.2	89.2(1.3)	3.4(0.001)	79.7(1.3)	3.0(0.001)
Seasonal	313.0	60.5(7.6)	3.0(0.004)	51.5(7.2)	2.5(0.004)
300-Day	300.0	52.4(7.2)	1.4(0.002)	44.2(6.6)	0.7(0.002)
Seasonal	258.0	5.5(0.9)	1.4(0.001)	4.6(0.9)	1.1(0.001)
Semiannual	182.6	0.5(0.6)	−0.1(0.000)	1.0(0.6)	−0.4(0.000)
4-Month	120.0	6.8(2.3)	0.6(0.003)	8.1(2.3)	0.8(0.003)
90-Day	90.0	0.9(0.3)	0.0(0.001)	0.3(0.3)	0.1(0.001)
Bimonthly	60.0	0.6(0.2)	−0.0(0.001)	0.2(0.2)	−0.0(0.001)
Monthly	30.0	0.1(0.1)	−0.0(0.001)	0.1(0.1)	0.0(0.001)
Triweekly	22.0	0.0(0.1)	−0.0(0.001)	0.0(0.1)	−0.0(0.001)

Values within the parentheses are the calculated standard deviations using variance propagation from the estimated errors of the harmonic coefficients

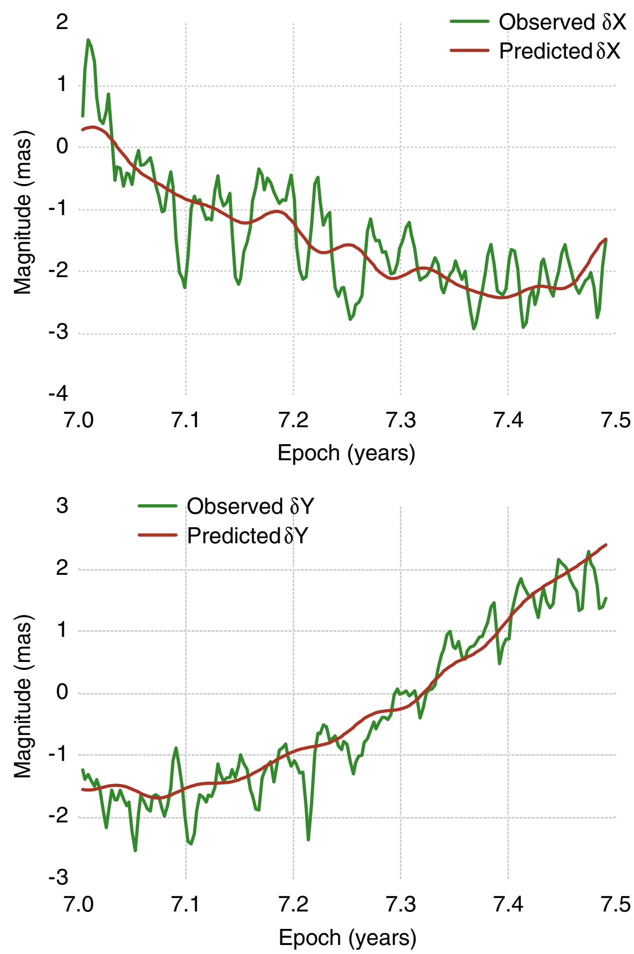

Fig. 9 Serially differenced daily PM data, the corresponding difference, and predicted PM components are displayed. Both series start at the ending epoch of the earlier series used during the estimation of the time-varying harmonic coefficients, and their epochs are centered at 31/12/1999, at the origin of the 14-year series

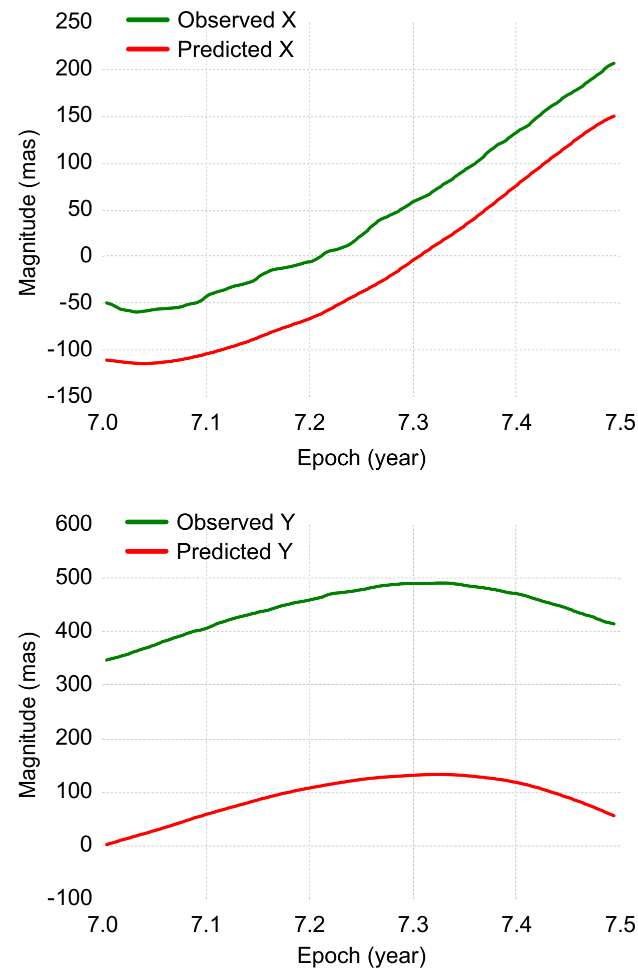

Fig. 10 Observed PM data and the predicted PM components. The series start at the ending epoch of the series used during the estimation of the time-varying harmonic coefficients and their epochs are centered at 31/12/1999, at the origin of the 14-year series

A time series of 6 months length, which is not included in the solution, was used to assess how well the solutions can predict. The mathematical model of the differenced observation Eq. (16) was evaluated at daily intervals using the estimated harmonic coefficients. Figure 9 shows the observed differences and the corresponding predicted values. Both series start at the ending epoch of 14-year series whose epochs are centered at 31/12/1999, i.e. they continue where the earlier series ended. The rms prediction errors for the serially differenced daily PM components are 0.5 and 0.4 mas for a 6-month ahead prediction. However, the predictions of the daily pole positions are of more interest.

The variable amplitude mathematical model (4) was used to predict the 6 months PM data. Since the intercept and slope information are not available from the solution, the observed and predicted values, as displayed in Fig. 10 are different from each other by almost constant offsets. The rms prediction error is about 3.6 and 3.8 mas for the x and y components if their systematic offsets were known precisely. In the absence of this information, the prediction can be carried in a number of ways:

1. The average residual values from the 14-year model can be used in the predictions. They represent constants because of the lump-sum unmodeled effects in the time-variable harmonic coefficient model. This approach gives 16.7 and 15.2 mas rms error for the 6-month ahead predictions for the PM components.
2. An estimate of the systematic offsets between the predicted and the observed PM values can be calculated for example, by averaging the difference of the first week's predicted and observed values. Because the averaging is based on most recent values, they are more representative of the unmodeled effects for the upcoming days as compared to the previous approach. The rms prediction errors, in this case, reduced

to 3.7 and 9.0 mas for the x and y components, respectively.

3. The following simple one-step-ahead predictor, which is an extension of (8), can also be used for forecasting as an estimate of the autoregressive disturbances not for the whole 6-month ahead but one day at a time for 6 months:

$$\hat{e}_{t+1} = \hat{\rho}\,\hat{e}_t \tag{17}$$

where $\hat{e}_t$ is the prediction error of an earlier day. The first order correlation coefficients of the series are very close to unity because of the very short periodic unmodeled tidal effects in the data. Therefore, the rms prediction error for one-day ahead prediction is almost the same as the earlier rms prediction errors for the serially differenced daily PM components that are 0.5 and 0.4 mas.

6 Conclusion

This study shows that the PM components can be modeled effectively using time varying harmonic coefficients and by differencing observation equations to eliminate autoregressive disturbances and resulting in sub mas level of measurement precision. The new model enables detection of closely spaced seasonal and interannual wobbles and accounts for the *unmodeled* long periodic effects through their lump sum effects estimated from the residuals.

The model's performance was verified using independent data, which indicated that the new model can also predict up to 6 months ahead with a few mas rms accuracy (for different approaches) including down to sub mas accuracy for a simple one-step-ahead prediction solution. The new model and solution approach can be used effectively to monitor global changes that excite polar motion and for better understanding their origins and their time evolution.

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