Tailored Datum Transformation Model for Locally Distributed Data

By Hüseyin Bâki Iz[1] and Yong-qi Chen[2]

ABSTRACT: A seven-parameter datum transformation model is constructed for transformations between the World Geodetic System (WGS84) datum and the Hong Kong 1980 local datum. The solution is tailored by biasing the height information in the WGS84 system to obtain better agreement with the Hong Kong datum height values. The influence of the tailoring process on the accuracy of the model parameters is assessed.

INTRODUCTION

Hong Kong (1980) (HK80) geodetic datum based on the International Hayford (1910) reference ellipsoid was adopted in 1978–1979. In 1990, a territory-wide observation on a network of 15 stations (12 of which are at existing trigonometric stations) was carried out using global positioning system (GPS) and Doppler satellite techniques. This survey provided a link between the local HK80 geodetic datum and the global WGS84 datum (SMO 1995).

Today, the connection between the WGS84 and the HK80 datums is established through a four-parameter similarity transformation between the two grid systems (WGS84 user grid and HK80 grid). These grids are generated by projecting the latitude and longitude information on both datums to a plane using transverse mercator projection. The four transformation parameters—two for the shift, one for the scale, and one for the plane rotation—have been obtained from a least-square adjustment of 12 control points whose coordinates are projected onto their corresponding grid coordinate systems (SMO 1993, 1995).

Because the Hong Kong Territories cover an area of approximately 50 km by 50 km, the existing plane transformation is sufficiently adequate for most mapping and surveying activities in the region. Nevertheless, increasing popularity of the GPS applications in the region recently prompted a virtual need for a three-dimensional formulation that is motivated by the familiarity of different user groups with this transformation. The fact that most of the existing software readily accepts the parameter of a three-dimensional transformation has also contributed to this virtual need.

In this study, we establish a seven-parameter transformation model between the global WGS84 datum and the local HK80 datum coordinate frames. This activity, however, is constrained by several factors. First, the model results are required to be in conformity with the existing two-dimensional transformation model results for practical reasons. Second, the datum ellipsoidal heights are possibly affected by a systematic offset introduced by the GPS

---

[1]Assoc. Prof., Dept. of Land Surv. and Geo-Informatics, The Hong Kong Polytechnic Univ., Hung Hom, Kowloon, Hong Kong.

[2]Prof., Chair, and Head, Dept. of Land Surv. and Geo-Informatics, The Hong Kong Polytechnic Univ., Hung Hom, Kowloon, Hong Kong.

Note. Discussion open until July 1, 1999. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on October 7, 1997. This paper is part of the ***Journal of Surveying Engineering***, Vol. 125, No. 1, February, 1999. ©ASCE, ISSN 0733-9453/0001-0025–0035/$8.00 + $.50 per page. Paper No. 16766.

control stations connected to the WGS84 via Doppler observations dating back to 1990. The model development problem is further complicated by the distribution of the available stations in a small area (50 km by 50 km); thereby, resulting in a poor geometry for the separability of the transformation parameters. These problems are addressed in the following sections.

MATHEMATICAL MODEL: SEVEN PARAMETER DATUM TRANSFORMATION

We consider a seven-parameter rigid body transformation model

$$\mathbf{x}_{WG} = \mathbf{c} + \mu\mathbf{R}\mathbf{x}_{HK} \tag{1}$$

where μ is a scalar denoting the scale factor; $\mathbf{R}$ is the 3×3 rotation matrix; $\mathbf{x}_{WG}$ and $\mathbf{x}_{HK}$ and $\mathbf{c}$ are the 3×1 position and the shift vectors of a collocated station in two different datums, i.e.

$$\mathbf{x}_{WG} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad \mathbf{x}_{HK} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{HK}, \quad \mathbf{c} = \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \end{bmatrix} \tag{2a–c}$$

Subscripts *WG* and *HK* indicate that the corresponding vectors refer to the WGS84 and the HK80 frames, respectively. Previous studies reported close agreement between the HK80 and the WGS84 frames (Mok 1992); therefore, the rotation matrix can be expressed in terms of differential rotations as (Rapp 1980)

$$\mathbf{R} = \begin{bmatrix} 1 & d\alpha_3 & -d\alpha_2 \\ -d\alpha_3 & 1 & d\alpha_1 \\ d\alpha_2 & -d\alpha_1 & 1 \end{bmatrix} = \mathbf{I} + \mathbf{dR} \tag{3}$$

where $\mathbf{I}$ is the identity matrix and

$$\mathbf{dR} = \begin{bmatrix} 0 & d\alpha_3 & -d\alpha_2 \\ -d\alpha_3 & 0 & d\alpha_1 \\ d\alpha_2 & -d\alpha_1 & 0 \end{bmatrix} \tag{4}$$

Substituting (4) and (3) into (1), and considering small rotation angles and a differential correction to the unit scale factor, namely, $\mu = 1 + d\mu$ then

$$d\mu d\alpha_i \sim 0 \Rightarrow d\mu\mathbf{dR} = \mathbf{0},\ i = 1, 2, 3 \tag{5}$$

and we finally obtain,

$$\mathbf{x}_{WG} - \mathbf{x}_{HK} = \mathbf{c} + (d\mu\mathbf{I} + \mathbf{dR})\mathbf{x}_{HK} \tag{6}$$

In this expression, the translation vector can also be expressed in a linearized form through which the corrections to the translation vectors are computed iteratively (Mikhael and Ackerman 1976; Hofman-Wellenhof et al. 1994). However, in this case, solving the translation parameters directly together with the other parameters does not harm the solution because the design matrix is independent of the translation parameters. Moreover, the sizes of the translation vector components are less than a few hundred meters.

Eq. (6) can be rearranged and expressed as

$$\mathbf{x}_{WG} - \mathbf{x}_{HK} = \mathbf{A}\mathbf{p} \tag{7}$$

where the design matrix $\mathbf{A}$ and the parameter vector $\mathbf{p}$ are given by

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & x_1 & 0 & -x_3 & x_2 \\ 0 & 1 & 0 & x_2 & x_3 & 0 & -x_1 \\ 0 & 0 & 1 & x_3 & -x_2 & x_1 & 0 \end{bmatrix} \tag{8}$$

$$\mathbf{p}^T := [c_1 \quad c_2 \quad c_3 \quad d\mu \quad d\alpha_1 \quad d\alpha_2 \quad d\alpha_3]^T \tag{9}$$

Note that if the Cartesian coordinates of a station in both datums are available then the left-hand side of (7) can be calculated using a set of collocated station coordinates given in both datums. The functional model is, nevertheless, not very suitable for numerical computations. The design matrix is composed of ones and coordinate values that have large magnitudes causing high condition numbers for the normal equations. Leick and van Gelder (1975) give an alternative well-conditioned model in which the solution is referenced to the centroid of the station coordinates. This application results in identical rotation and scale parameters as the one discussed above and the translation parameters, which are different, can be obtained after an additional transformation. We have obtained numerical solutions using both functional models. They have not showed any numerically significant discrepancy. Nevertheless, we still operated on the existing model because it is more transparent for the error analysis we will formulate in the next section.

In (7), the station coordinates are also linearly related to the transformation parameters. Now, this functional model can be deployed in a statistical model to estimate the transformation parameters.

STATISTICAL MODEL

The observations, Cartesian coordinates in both datums, suggest that the statistical model is based on a set of condition equations with unknown parameters. That is

$$\mathbf{v}_{WG} - \mathbf{v}_{HK} - \mathbf{Ap} - (\mathbf{x}_{WG} - \mathbf{x}_{HK}) = 0 \tag{10}$$

where $\mathbf{v}_{WG}$ and $\mathbf{v}_{HK}$ are the corresponding observation error vectors (residuals) and the expression in parentheses gives the misclosure vector. However, this model can also be equivalently represented by a set of linear observation equations as

$$\mathbf{y} = \mathbf{Ax} + \mathbf{u} \tag{11}$$

where $\mathbf{y}$ is the $3n \times 1$ vector of quasi-observations; and n = number of stations

$$\mathbf{y} = \mathbf{x}_{WG} - \mathbf{x}_{HK} \tag{12}$$

$\mathbf{A}$ is the $3n \times m$ design matrix as defined above for a single point; m = number of unknown parameters, which is seven in this model. The $3n \times 1$ vector of disturbances $\mathbf{u}$ is defined as

$$\mathbf{u} := \mathbf{v}_{WG} - \mathbf{v}_{HK} \tag{13}$$

The disturbances are assumed to have the following distributional properties

$$E(\mathbf{v}_{WG}) = E(\mathbf{v}_{HK}) = \mathbf{0}, \Rightarrow E(\mathbf{u}) = \mathbf{0}, \text{ and for } i = 1, \ldots, 3n \tag{14a}$$

$$E(\mathbf{v}_{WG}\mathbf{v}_{WG}^T) = \text{diag}(\sigma^2_{WG_i}), \; E(\mathbf{v}_{HK}\mathbf{v}_{HK}^T) = \text{diag}(\sigma^2_{HK_i}) \tag{14b}$$

$$E(\mathbf{v}_{WG}\mathbf{v}_{HK}^T) = \mathbf{0}, \Rightarrow E(\mathbf{u}\mathbf{u}^T) = \sigma^2 \text{ diag}(\sigma_{WG_i}^2 + \sigma_{HK_i}^2) = \Sigma \qquad (14c)$$

This representation offers advantages as compared with the condition equation formulation. Some of the well-known analysis techniques for the observation equations can readily be used without resorting to additional derivations in the case of condition equations. Although the weighted least-squares method is the obvious candidate to obtain a solution to the above statistical model, the design matrix described in (8) exhibits some peculiarities that indicate potential problem areas if the normal equations are used to estimate the model parameters. When the coordinates in the observation equations are expressed in kilometers, the ratio of the maximum to minimum singular values for the design matrix is about 10^6. Furthermore, scaling of the units does not change this number. This value will be squared when the normal equations are formed, leading to eigenvalue ratios larger than 10^{12}, which may create undesirable computational round-off errors. Under these circumstances, we choose to obtain a least-square solution by directly solving the observation equations through singular value decomposition of the observation equations to prevent potential numerical instabilities. The solution in this case involves singular value ratios on the order of 10^6.

We first normalize the observations with the variance/covariance matrix of the quasi-observables introduced above

$$\bar{\mathbf{y}} = \bar{\mathbf{A}}\mathbf{x} + \bar{\mathbf{u}} \qquad (15)$$

where

$$\bar{\mathbf{y}} := \Sigma^{-1/2}\, \mathbf{y} \qquad (16a)$$

$$\bar{\mathbf{u}} := \Sigma^{-1/2}\, \mathbf{u} \qquad (16b)$$

As is well known, there exist orthogonal matrices, **U** and **V** of dimension $m \times 3n$ and $m \times m$, respectively, such that

$$\mathbf{U}\bar{\mathbf{A}}\mathbf{V}^T = \Lambda \qquad (17)$$

where

$$\Lambda = \text{diag}(\lambda_1, \lambda_2, \ldots\ \lambda_m) \qquad (18)$$

Writing

$$\mathbf{z} = \mathbf{U}\bar{\mathbf{y}}, \quad \gamma := \mathbf{V}\mathbf{x}, \quad \nu := \mathbf{U}\bar{\mathbf{u}} \qquad (19a\text{–}c)$$

we obtain

$$\mathbf{z} = \Lambda\gamma + \upsilon \qquad (20)$$

The least-squares solution to the above transformed sets of equations is given by

$$\hat{\gamma}_i = \frac{\mathbf{z}_i}{\lambda_i},\ i = 1, \ldots, m \qquad (21)$$

The variance/covariance matrix of the estimated transformed parameters, $\hat{\gamma}$, is calculated from

$$E(\hat{\gamma}_i = \gamma_i)^2 = \frac{\hat{\sigma}^2}{\lambda_i^2} \quad (22)$$

where $\hat{\sigma}^2$ is the a posteriori variance of unit weight. The circumflex on the variables indicates that they are estimated quantities.

DISCUSSION ON HEIGHT INFORMATION IN WGS84 AND HK80 DATUMS

Since the Hong Kong territories cover only a small area (approximately 50 km by 50 km, (Fig. 1), information about local geoid undulation has never been a critical issue for surveying and mapping practice. In Hong Kong, all heights and levels on land refer to a level called Hong Kong Principal Datum (HKPD). At this location, the mean sea level is determined to be approximately 1.23 m above HKPD using observation records from 19 years (1965–1983) of automatic tide gauge measurements. However, no special efforts have been made to unify this information with the origin of the primary geodetic network established in Hong Kong. Moreover, the so-called zero trigonometric point previously established to provide orientation and to fix the origin of the primary geodetic network through astronomical observations, is no longer accessible. Although local gravimetric information for this region has been available for some time, this information was not used to construct a detailed local geoid model.

Despite these setbacks, the unavailability of geoid undulations in the Hong Kong datum does not pose a serious problem to achieving a datum transformation solution. The tide gauge station to which the Hong Kong datum is connected is located within a few kilometers of the old zero trigonometric point, whose orthometric height is also tied to the vertical datum. Therefore, for all practical purposes, we may assume that the geoid and the ellipsoid of

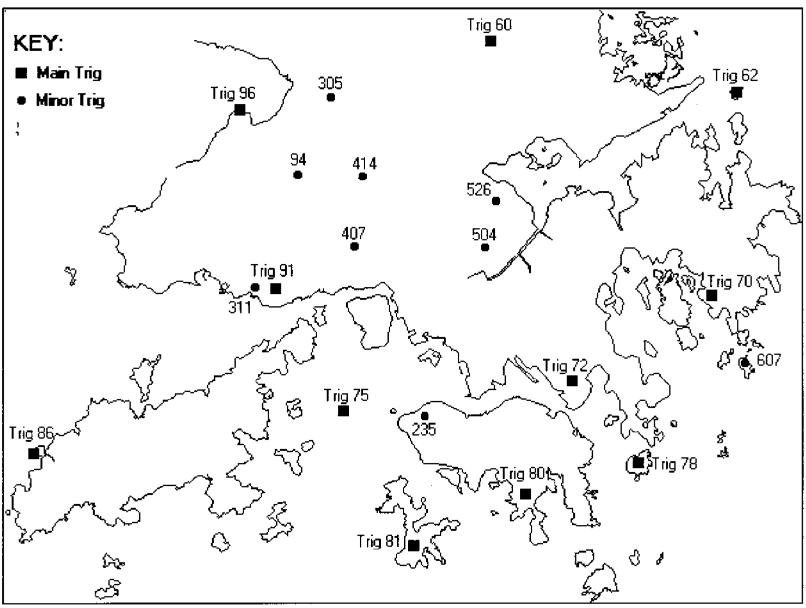

FIG. 1. Distribution of 20 Trigonometric Stations Used in Solution

the Hong Kong datum coincide at the zero trigonometric point of the primary horizontal control network.

Because the Hong Kong territories cover only a small area, we do not expect a lot of variability in the magnitude of the geoid undulations in the region. Hence, we may assume that all the leveled heights in the Hong Kong vertical datum—after being corrected for the principal datum offset, which is determined by almost two decades of tide gauge observations—will refer to orthometric heights. Orthometric height corrections (orthometric excess) for the leveled heights can also be neglected for the same reason.

Given the above discussion and considering the availability of accurate orthometric height information through precise leveling, we have now two alternative ways to obtain geoid undulations in the WGS84 datum. We can either compute them from the existing WGS84 harmonic model or from the known orthometric heights and the GPS observed WGS84 ellipsoidal heights

$$N_{WGS84} \cong h_{WGS84} - H \tag{23}$$

$$N_{Harm} \cong h_{Harm} - H \tag{24}$$

where N_{WGS84} is the geoid undulation referring to the WGS84 datum; H = orthometric height; h_{WGS84} is is the ellipsoidal (geometric) height obtained from GPS measurements; and h_{Harm} is the ellipsoidal height given by

$$h_{Harm} \cong H + N_{Harm} \tag{25}$$

where N_{Harm} is the geoid undulation referring to WGS84 system that is calculated from the associated harmonic gravity model of the WGS84 datum (EGM80). The difference, (24) − (23)

$$\Delta N = N_{Harm} - N_{WGS84} \cong h_{Harm} - h_{WGS84} \tag{26}$$

shown in Fig. 2, indicates marked systematic, but smooth discrepancies (minimum 1.41 m; maximum 1.71 m) between GPS implied datum ellipsoidal heights and the ones computed from the harmonic model. This offset, on one hand, can be attributed to the inaccuracy of the harmonic model used in the computations, which is reported to be 3–6 m at some locations. On the other hand, the results from a 1990 territory-wide survey of a network of 15 stations can also be blamed for this discrepancy. In the old adjustment, the origin of the GPS network was fixed to the coordinates provided by the Doppler observations. Therefore, the determined GPS coordinates are also affected by the uncertainties in the adopted Doppler coordinates of the origin station. We are currently processing continuous GPS data obtained by a collocated first-order trilateration station in Hong Kong, together with data from several International GPS Service stations in the region. Preliminary coordinate estimates in the WGS84 frame obtained from the IERS Terrestrial Reference Frame connection show an over 3 m discrepancy in height.

Given these equally likely potential error sources, we choose to obtain an internally consistent transformation rather than a correct one. This is actually one of the requirements we imposed on ourselves in this study in order to maintain continuity between the coordinates of both models (i.e., the existing two-dimensional and three-dimensional).

Assume for the moment that the ellipsoidal heights in the WGS84 datum computed from the harmonic model are correct. Then, we can apply corrections given by (26) to the undulations in the WGS84 datum (equivalent to replacing the ellipsoidal height information with the computed height information) and proceed the estimation of transformation parameters as we have

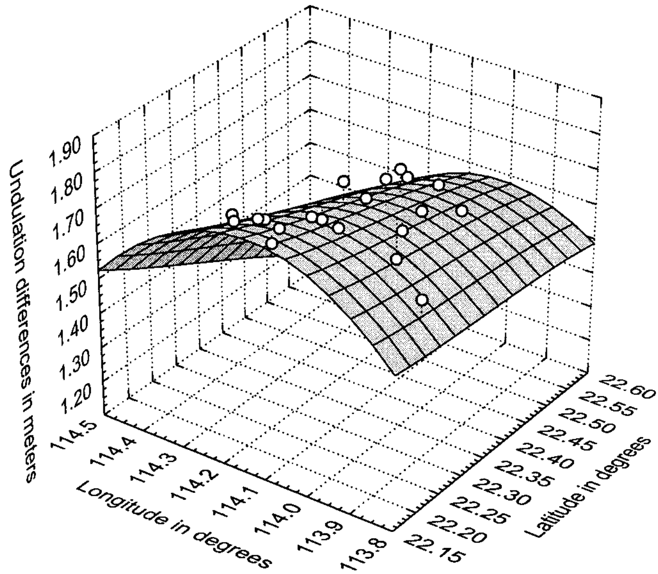

FIG. 2. Undulation Differences Modeled Using Bicubic Model (Refer to Appendix I for More Information)

discussed previously. Modeled undulation differences can be applied as corrections to any GPS data dependent on the existing GPS station coordinates as part of the transformation process.

If the computed values are off from their true values, (26) is then an estimate of the inconsistency between the undulation values obtained from two different approaches. In this case, replacing height values in the WGS84 datum via computed height values (by means of undulation corrections) will remove this inconsistency. However, the problem in this case is that we can no longer claim that the results are accurate, but we can estimate the impact of the height uncertainties on the estimation of parameters. This is the topic of the next section.

MODELING INFLUENCE OF INDUCED ELLIPSOIDAL HEIGHT UNCERTAINTIES

Consider the following well-known relationship between the ellipsoidal coordinates and the Cartesian coordinates of a point above the ellipsoid

$$\mathbf{x}_{WGS} = \begin{bmatrix} (N + h)\cos\phi\cos\lambda \\ (N + h)\cos\phi\sin\lambda \\ [N(1 - e^2) + h]\sin\phi \end{bmatrix} \quad (27)$$

where N is the prime vertical radius of curvature; h = ellipsoidal height; e is the first eccentricity; ϕ and λ are the ellipsoidal latitude and longitude, respectively; and $\mathbf{x}_{WGS}$ is the 3×1 Cartesian position vector. All quantities refer to the WGS84 datum. Any point error because of the corrections applied to the GPS heights, as we have discussed in the previous section, can be approximately represented as a constant over the region. Hence, from (23) a small change in ellipsoidal height can be expressed as

$$\bar{\mathbf{x}}_{WGS} = \mathbf{x}_{WGS} + \delta\mathbf{h} \tag{28}$$

with

$$\delta\mathbf{h} = \begin{bmatrix} \delta h \cos\phi \cos\lambda \\ \delta h \cos\phi \sin\lambda \\ \delta h \sin\phi \end{bmatrix}, \ |\delta\mathbf{h}| = \delta h \tag{29}$$

Rewriting (6), we can express the transformation model to include also the effect of height error as

$$\mathbf{x}_{WG} = \mathbf{c} + (1 + d\mu)\mathbf{R}\mathbf{x}_{HK} + \delta\mathbf{h} \tag{30}$$

Eq. (7) will now take the form

$$\mathbf{x}_{WG} - \mathbf{x}_{HK} = \mathbf{A}_1\mathbf{x}_1 + \mathbf{A}_2\mathbf{x}_2 \tag{31}$$

where

$$\mathbf{A}_2 = \frac{\delta\mathbf{h}}{\delta h} \tag{32}$$

$$\mathbf{x}_2 = \delta h \tag{33}$$

Because of the critical configuration (all the trigonometric stations to be used in the solutions are confined to a 50 km by 50 km region) and high correlation between the scale and height parameters, we will not solve for this effect in our model. Instead, we will generate a statistic to evaluate the effect of its omission on the estimated transformation parameters. We will use the mean square error (MSE) matrix for this purpose.

Given the well-known definition of the MSE, it can be shown after some matrix manipulations, using (7)–(14) and (26)–(31) that

$$\mathrm{MSE}(\hat{\mathbf{x}}_1) := E(\hat{\mathbf{x}}_1 - \mathbf{x})(\hat{\mathbf{x}}_1 - \mathbf{x})^T$$

$$= (\mathbf{A}_1^T \Sigma^{-1} \mathbf{A}_1) + (\mathbf{A}_1^T \Sigma^{-1} \mathbf{A}_1)^{-1} \mathbf{A}_1^T \Sigma^{-1} \mathbf{A}_2 \delta h^2 \mathbf{A}_2^T \Sigma^{-1} \mathbf{A}_1 (\mathbf{A}_1^T \Sigma^{-1} \mathbf{A}_1)^{-1} \tag{34}$$

Because the first term in the above relationship is the variance/covariance matrix of the least-square solution when no systematic height error is introduced into the solution, the second term represents the omission of the systematic height error that is not included in the solution.

SOLUTION AND ANALYSIS OF RESULTS AND CONCLUSIONS

A solution using the statistical model described by (1)–(7) was obtained. We used, as observations for the transformation model, the Cartesian coordinates of the 20 trigonometric stations in WGS84 and HK80 datum shown in Fig. 1 calculated from the ellipsoidal coordinates. In these computations, the ellipsoidal heights in the WGS84 datum were replaced with the ones obtained from the harmonic model and known orthometric heights of the stations following the discussion made in the previous section. The observational noise level for all the coordinates were assumed to be 1 and 5 cm for the Hong Kong and WGS84 coordinated, respectively. The a priori variance of unit weight was initially assumed to be one, and then replaced by the a posteriori variance of unit weight after the adjustment.

The uncertainties of the estimated transformation parameters were also computed using (34) and displayed in Table 1, together with the influence of a postulated 5 m height error. These results show that the uncertainties of the estimated shift and the rotation parameters are mostly influenced by the geometry of the problem (as indicated by the variance/covariance matrix of the solution). Overall, the effect of height error on the solution is less than the noise effect except for the systematic height error that dominates the uncertainty of the estimated scale parameter. Note that there is no surprise about these findings since the relationship between the height and the scale is well known in geodetic applications. We also evaluated the influence of the height error for a range of values (0–5 m) and displayed the results in Fig. 3.

These results show a linear dependency between the height error and the error of the estimated scale parameter (please note that the vertical axis of the plot is in logarithmic scale).

We discussed that the modifications we have made on the ellipsoidal height will ensure to reduce the discrepancy of the solution from the existing height and four-parameter transformation. We computed the positions of seven trigonometric stations that are not used in the solution using transformation mod-

TABLE 1. Estimated Parameters for Transformations from HK80 Datum Coordinates to the WGS84 Datum Coordinates

Parameters (1)	Estimated values (2)	Total error (3)	(5 m) Height error contribution (4)
Δx_1	−165.207	1.960	0.017
Δx_2	−297.574	1.772	0.006
Δx_3	−132.642	2.354	0.015
$d\mu$	−0.6713E-7	8.2236E-7	7.8525E-7
$d\alpha_1$	0.96319	0.07596	0.00001
$d\alpha_2$	2.61013	0.05797	0.00023
$d\alpha_3$	1.49050	0.006312	0.00038

Note: Shift values are in meters whereas differential rotation angles are in arc seconds.

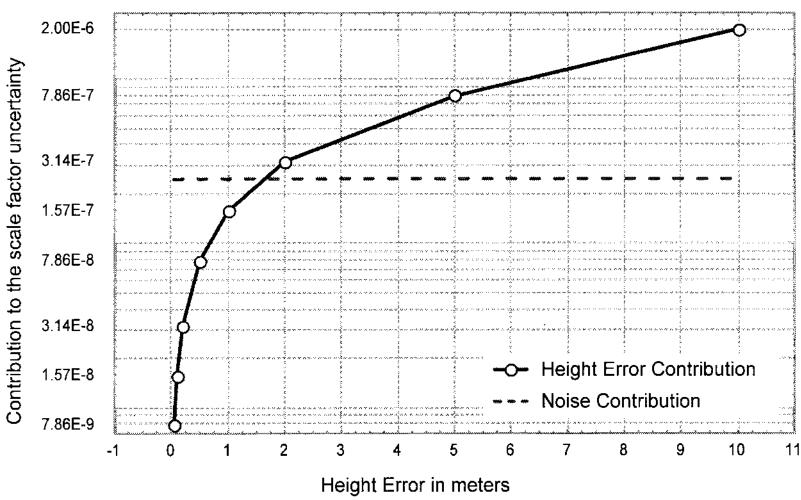

FIG. 3. Contribution of Systematic Height Error on Scale Factor Uncertainty

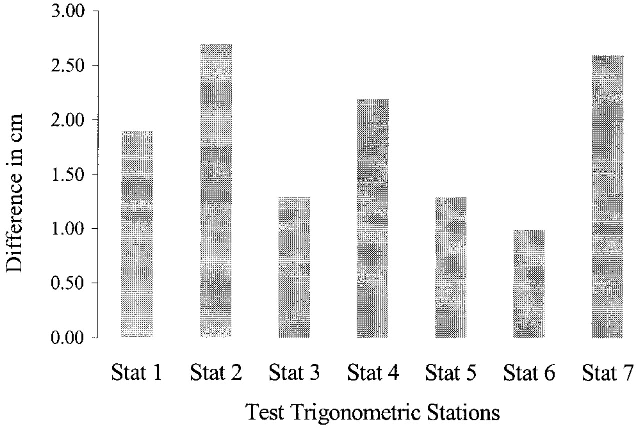

FIG. 4. Difference between Test Trigonometric Station Positions as Predicted by 2D and 3D Transformation Models to WGS84 Datum

els in the WGS84 datum to quantify the agreement between the two systems (four-parameter and height and seven-parameter transformations).

Fig. 4 shows the difference between two transformation model results for transformed positions of stations in the Hong Kong datum. The differences are less than 3 cm and most of it can be attributed to the use of a larger number of trigonometric stations in the seven-parameter transformation model (thereby a better solution with more and better data). Earlier four-parameter transformation model included only 12 trigonometric stations, whereas there are eight more new stations in the current solution.

APPENDIX I. CALCULATION OF UNDULATION CORRECTIONS

Let ϕ and λ denote, respectively, the latitude and the longitude of a station for which the geoid undulation correction will be interpolated, and let ϕ_N, ϕ_S and λ_N, λ_S represent the known coordinates for normalizing the latitude and longitude. Then, the interpolation formula for calculating the correction ΔN (in meters) to the WGS84 geoid undulations, is given by

$$\Delta N = 1.67061 - 0.14784\lambda_N^2 - 0.06918\phi_N^3 - 0.13286\phi_N\lambda_N^2$$

where

$$\phi_0 = 22°.367,\ \phi_S = 0°.175,\ \lambda_0 = 114°.111,\ \lambda_S = 0°.255,$$

$$\lambda_N = \frac{\lambda - \lambda_0}{\lambda_S},\ \phi_N = \frac{\phi - \phi_0}{\phi_S}$$

This model is obtained through a least-squares fitting procedure to the undulation differences using a high order polynomial model whose significant coefficients are determined using a hypothesis testing procedure.

ACKNOWLEDGMENTS

We are grateful to an anonymous reviewer whose careful comments considerably improved the readability of the manuscript. Part of this study is supported by the Area of Excellence Project and by Internal Research Grant 0351/615 of The Hong Kong Polytechnic University.

NIMA of the United States kindly provided the spherical harmonic model of the WGS84 datum. The information about the trigonometric station values in both datums is provided by the Survey and Mapping Office Land Department of Hong Kong.

APPENDIX II. REFERENCES

Hofman-Wellenhof, B., Lichtenegger, H., and Collins, J. (1994). *GPS theory and practice*, 3rd Ed., Springer, Wien, Germany.

Leick, A., and van Gelder, B. H. W. (1975). ''On similarity transformation and geodetic network distortions based on Doppler satellite observations.'' *OSU Rep. 235*, Department of Geodetic Science, Ohio State University, Columbus, Ohio.

Mikhael, E. M., and Ackermann, F. (1976). *Observations by least squares*. Harper & Row, New York.

Mok, E. (1992). ''A model for the transformation between satellite and terrestrial networks in Hong Kong.'' *Survey Rev.*, 31, 244.

Rapp, R. (1980). *Geometric geodesy class notes.* Department of Geodetic Science, Ohio State University, Columbus, Ohio.

S.M.O., Land Department. (1993). ''Conversion of coordinates between World Geodetic System (WGS84) and Hong Kong 1980 Grid System (HK80).'' Hong Kong Lands Department Publication, Hong Kong.

S.M.O., Lands Department. (1995). ''Explanatory notes on geodetic datums in Hong Kong.'' Hong Kong Lands Department Publication, Hong Kong.