The accuracy of earth orientation parameters estimated from VLBI data using variate-differencing

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Abstract

In this study we developed a bootstrapping approach using a variate-differencing algorithm to check the formal error estimates of the earth orientation parameters using earth orientation parameters themselves.

We used the EO parameters reported by the GSFC VLBI Solution GLB1122 - June 1999 and their formal errors in this process. We found that the average formal error estimates and the error estimates from the variate-differencing are in close agreement with each other over the entire time span of the polar motion series. However, the variate-differencing errors are approximately 4.5 times larger than the formal errors of the earth rotation series. We also observed that the errors for the polar motion and earth rotation series exhibit pronounced deviations from the ones estimated by variate-differencing at local scales, which may be due to the sensitivity of the variate-differencing algorithm to the varying span of the data or failure of the algorithm with increasing precision of VLBI measurements.

Introduction

VLBI group of the NASA's Goddard Space Center operates a program in Very Long Baseline Interferometry (VLBI) data acquisition and analysis to monitor changes in the orientation of the Earth on a regular basis (Ma, et al, 1998). In the interim, the Earth Orientation (EO) results are obtained from long duration experiments in a multi-parameter least squares adjustment for polar motion (Figure 1), for UT1 (Figure 3), and a multitude of model parameters.

Currently the most accurate method of determining the EO parameters at time resolution longer than a few days is VLBI that allows determination of polar motion parameters with a precision of 10^{-5} arcs and UT1 – UTC down to 10^{-6} s (Figures 2 and 4).

Although the multi-parameter least square solutions are extensively scrutinized for a calibrated solution, there are no other techniques and solutions which will verify the formal errors of these solutions. In this study, we developed a bootstrapping approach to obtain alternative estimates of the errors from the estimated polar motion components and UT1-UTC variations themselves using a variate-differencing algorithm and compared them to the formal errors reported by the least squares solution.

We used the EO estimated and their formal errors reported by the GSFC VLBI Solution GLB1122 -

June 1999 in this comparison.

Approach

Any types of series that are inferred from other observations are affected by *natural* changes. They may exhibit long term trends, periodicities (seasonality), systematicities that are aperiodic but can be represented by deterministic functions, interruptions (slips, or jumps or transitions) and processes that are stochastic in behavior over a prescribed interval. Inevitably, they also contain errors of different types which may be due to instrumental sources, inhomogeneity produced by the sampling and unmodeled model parameters.

For a given series, if any *natural* changes and systematic influences of environmental, sampling, or instrumental type are removed, there remains only the random variability in the series. In a well-calibrated system, the formal error estimates of the series and the remaining random variability obtained by removing the systematic effects are expected to be in close agreement.

Here, we used a variate-differencing algorithm where the nonrandom variability is readily eliminated by differencing the subsequent elements of the series leaving only the effects due to the random effects. This approach, besides its simplicity has also the ability to operate on certain type of non-stationary (in mean and variance) time series, which is not readily available through alternative approaches.

Variate-differencing

Consider a series that can be represented locally by a set of polynomials (systematic variations) with superimposed random elements. In this series the polynomial part can be eliminated by a sequence of successive differencing of the adjacent points. The differencing progressively reduces the order of the polynomial while increasing the variance of the residuals that are random in nature. This enhancing property of differencing can be used to derive an estimate for the variance representative for the whole series (Yule and Kendall, 1950, and Iz and Shum, 1998).

It can be shown that (ibid.) if e_t represents the series, and r is the r^{th} difference, then the variance of the series V_r is given by

$$V_r = \frac{\mathrm{var}(\Delta^r e_t)}{\binom{2r}{r}}$$

In the above expression, Δ denotes the differencing operator applied to the successive elements of the series. The numerator is given by the sum of the squares of the r^{th} differences. Suppose that the series are deterministic linear function of time then first difference of the series will give a constant for all time series eliminating the linear time varying part. In general, if the series consists of a k^{th} degree polynomial in time and a stationary random component, then the k^{th} differencing is stationary. It can be shown that the approach will also work for non-stationary autoregressive series with a random stationary component.

Observe that the above algorithm assumes that the entire series can be represented by polynomials (smooth functions), which can be achieved in our case by removing any other non-polynomial variation such as interruptions, such as slips or jumps from the data.

The application of the algorithm produces a single statistic representative of the random variations in the series, which can be compared with the formal errors that are available from the solution that generated the series.

Formal errors of the GSFC VLBI Solution GLB1122

We used the EO parameters estimated by the GSFC VLBI Solution GLB1122 - June 1999. Figure 1 shows the variations in the x and y pole components that are dominated by the Chandler wobble and annual oscillations.

In addition to Figure 2, the formal errors of the solution for the pole position components are also grouped into 40 sub-series. The averaged values of the formal errors in each sub-series are displayed in Figures 6, and 7. They clearly reflect the progressive improvements in the observational precision over time. Similarly, Figure 3 displays the UT1 – UTC differences and Figure 4, its errors. Figure 8 shows, similar to the PM series, the averaged values of the UT1-UTC differences in each series after the 1s-episodic adjustments are removed from the data. A 5-days sampling interval dominates all the series.

Observe that all the series are non-stationary in mean, because of the *natural variations* and in variance because of the progressive improvements in VLBI measurements over time.

Precision of EO parameters from formal error estimates and variate-differencing

We applied the variate-differencing algorithm directly to the polar motion and earth rotation series. Figure 5 shows that all the series converged rapidly up to the 7^{th} iteration then stabilized thereafter. Iterations beyond 15 do not contribute significantly to the final error estimates, which are listed in Table 1.

The error estimates as revealed by the variate-differencing and the average error of the solution formal errors are in close agreement with both polar motion components. Their ratios can be interpreted as a scale factor similar to the *a posteriori* *variance of unit weight* of the formal solution (weighted *Chi-square* divided by the degree of freedom). Given the seemingly unrelated nature of the two approaches, the ratio of 1.44 and 0.95 is remarkable. The ratio of 4.5 for the UT1-UTC series however, is markedly large. If this difference were due to the *a posteriori variance of unit weight* of the formal solution, we would have seen the same ratio for the PM components because the a posteriori variance applies to all solution errors simultaneously.

To further investigate this problem we have divided all the series into sub-series and applied the same approach as we did for the entire series to each sub-series.

Variate-differencing with sub-series

The formal errors of the solution for the EO parameters are grouped into 40 sub-series in such a way that each sub-series consists of 75 data points to remove the influence of the data density on the results. Their averaged values are displayed in Figures 6, and 7. The same figures also include the error estimates for each series obtained using variate-differencing algorithm with 15 iterations.

Again PM sub-series averaged formal errors and the variate-differencing errors are in close agreement up to 33^{th} sub-series. Discrepancies in the first couple of sub-series are due to the outlying solutions. Also, the average formal errors of the

UT1-UTC sub-series, once scaled by an amount of 4.5, are also in close agreement with their variate differenced counterparts for the first 33 subseries.

All EO series errors after the first 33 subseries progressively diverge in time. Although the sub-series contain the same number of data points, they do not span a constant period as can be seen from Figure 9. The frequency of network sessions that generate data point for the series in the reference frame solution progressively increases in time, until the middle of the entire series and then decreases in time for the remainder of the sub-series.

Because the variate-differencing algorithm has no means to account for the effect of time, we generated another variate-differencing solution in which the elements of each sub-series are weighted by a ratio inversely proportional to the square of the time span of data in each series. Figure 10, 11 and 12 show that the resulting error estimates are in better agreement with the average values of the formal errors in each subseries especially in the problem series. They nevertheless create problems in early sub-series.

At this point of the investigation, we can cite the following possibilities for the partial discrepancies and the overall agreement.

- The overall agreement indicated by the ratio of the errors (Table 1) estimated from two different sources for the PM series shows that the formal solution errors are well calibrated.

- The discrepancy by a factor of 4.5 between the earth UT1-UTC errors by the two sources can be attributed to the nature of the series that includes more erratic data than the smoother PM series. Variate-differencing algorithm is more effective in revealing the random behavior of the data in time than the formal errors, which are obtained somewhat independently, and locally from the earlier data points.

- The presence of progressively increasing error estimates for all series starting at the same time is intriguing. Although the changing frequency of the measurements can be a factor, it fails to fully answer the deviation. This problem can be attributed more to increasing precision of the VLBI measurements to a level that some of the periodic variations in the series become more pronounced and cause the variate-differencing algorithm to fail (for instance, it can be shown that the differencing algorithm will fail when there are periodic variations in the data with a period twice of the sampling interval(Yule and Kendall, 1950).

- A far fetched explanation is that perhaps the underlying natural changes in the data is getting noisier in time that can not be revealed by the formal errors of series elements which are not significantly serially correlated in the VLBI reference frame solution!

Acknowledgement

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References

Ma, C., et al., 1998, AJ 116 516-546.

Yule G.U, and M.G. Kendall, 1950: An introduction to the theory of statistics, C. Griffin pub. 14th ed.

Iz, H. Baki, and C.K. Shum, Sea level and GPS projects in Hong Kong and East China Sea, GLOSS Regional Sea Level Meeting, Taipei, Taiwan, July, 1998.

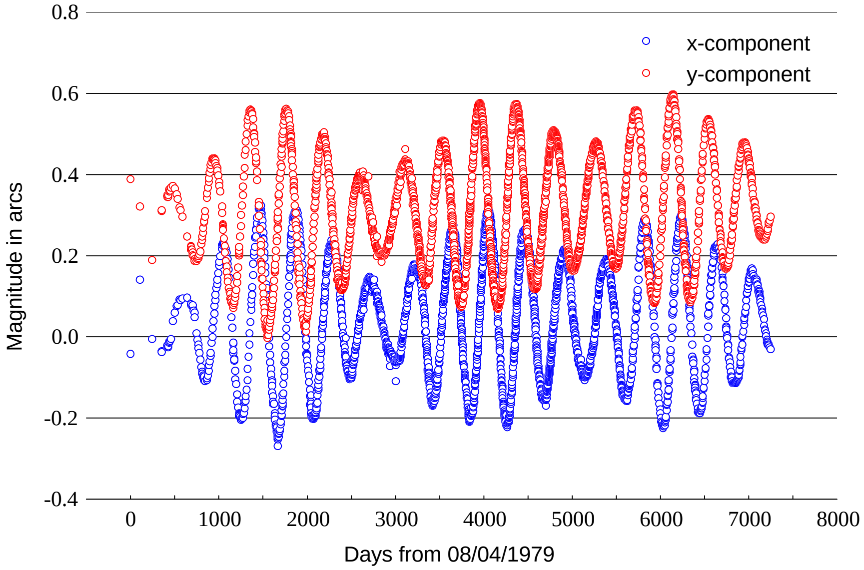

Figure 1. x and y components of the polar motion series estimated by the GSFC VLBI Solution GLB1122 - June 1999.

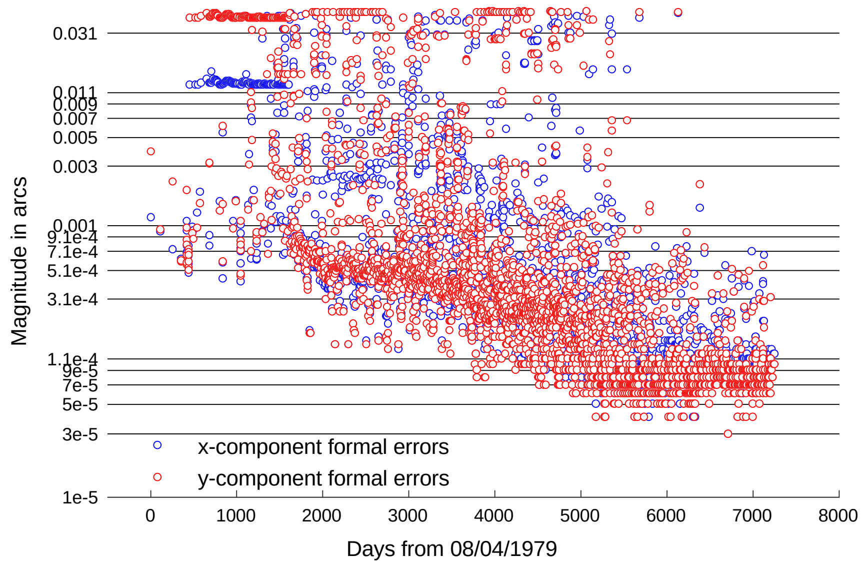

Figure 2. The formal errors (standard deviation) of the x and y components of the polar motion series estimated by the GSFC VLBI Solution GLB1122 - June 1999.

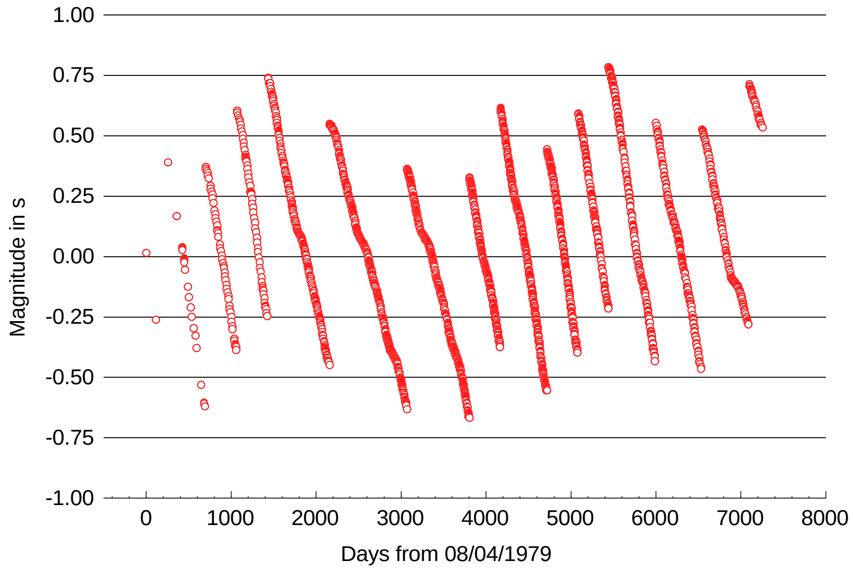

Figure 3. UT1 – UTC estimated by the GSFC VLBI Solution GLB1122 - June 1999.

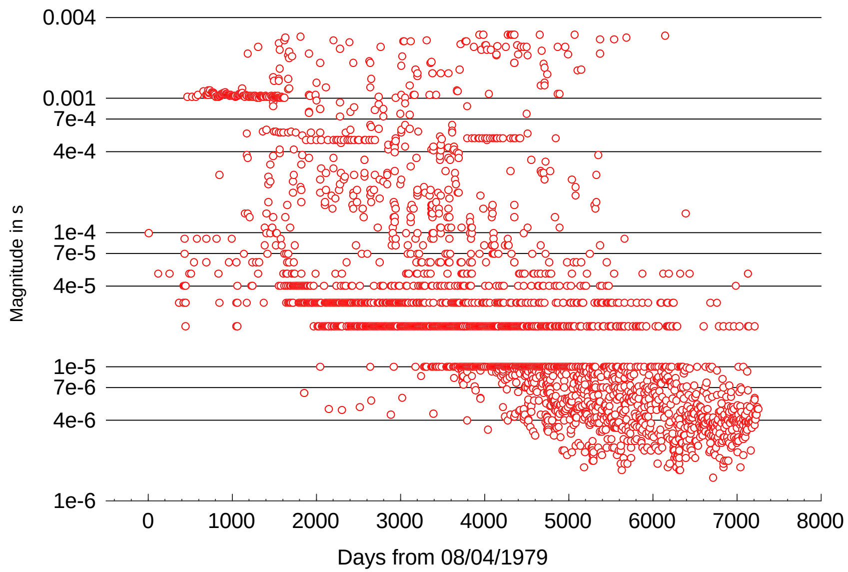

Figure 4. UT1 – UTC formal errors estimated by the GSFC VLBI Solution GLB1122 - June 1999.

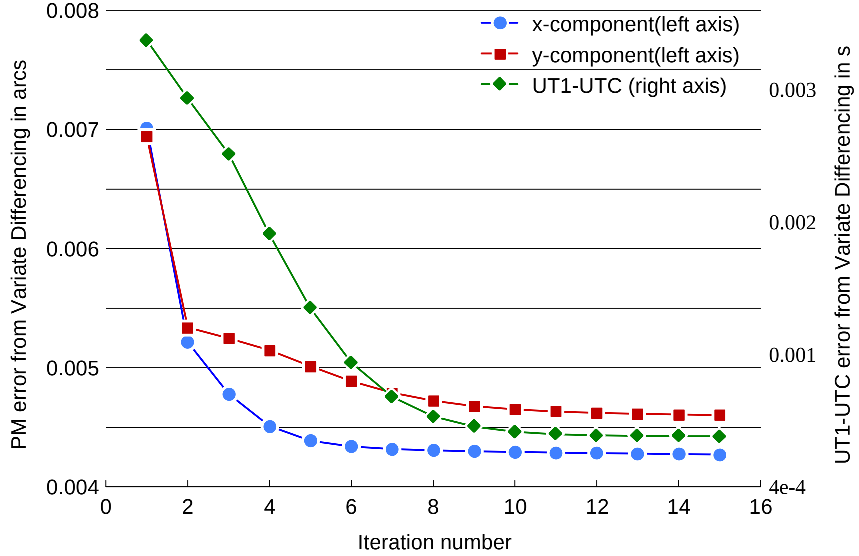

Figure 5. Convergence of the x and y polar motion and UT1-UTC series errors by variate differencing.

Table 1. Average errors from variate differencing and formal solution errors.

Series	Variate-differencing	Average of all formal errors	Ratio
x-comp.	0.0038 arcs	0.0027 arcs	1.44
y-comp.	0.0041 arcs	0.0043 arcs	0.95
UT1-UTC	0.00078 s	0.00017 s	4.59

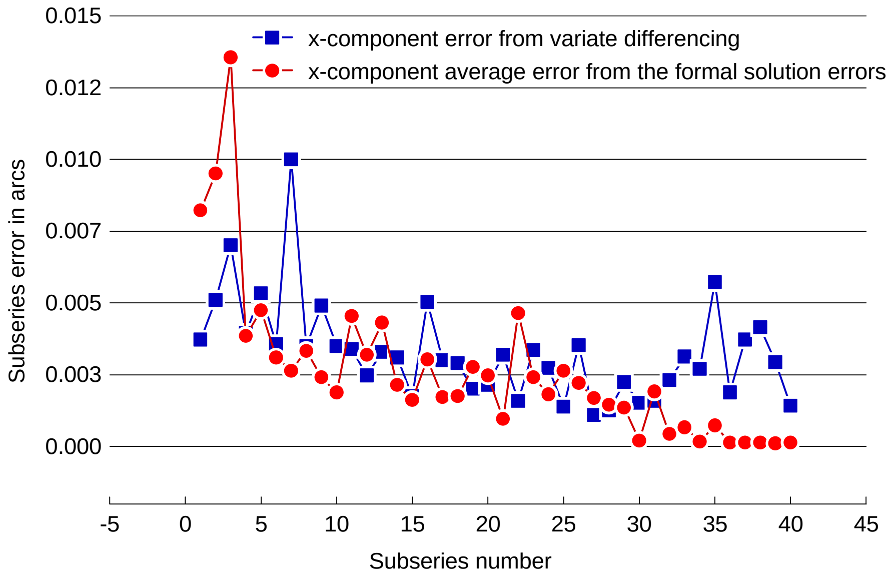

Figure 6. Polar motion x-component error estimated by the variate differencing for each sub series and the average formal errors of the subseries from the formal solution.

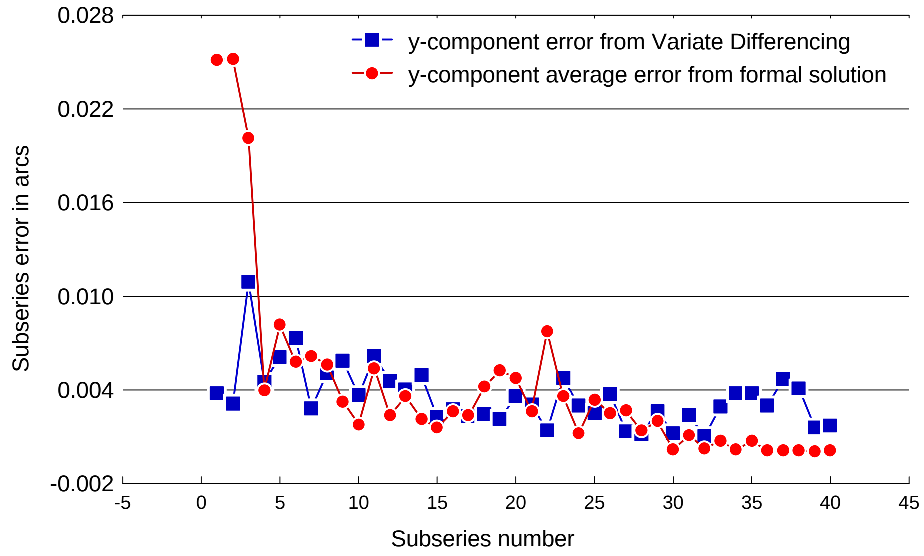

Figure 7. Polar motion y-component error estimated by the variate differencing for each subseries and the average of the formal errors of each subseries.

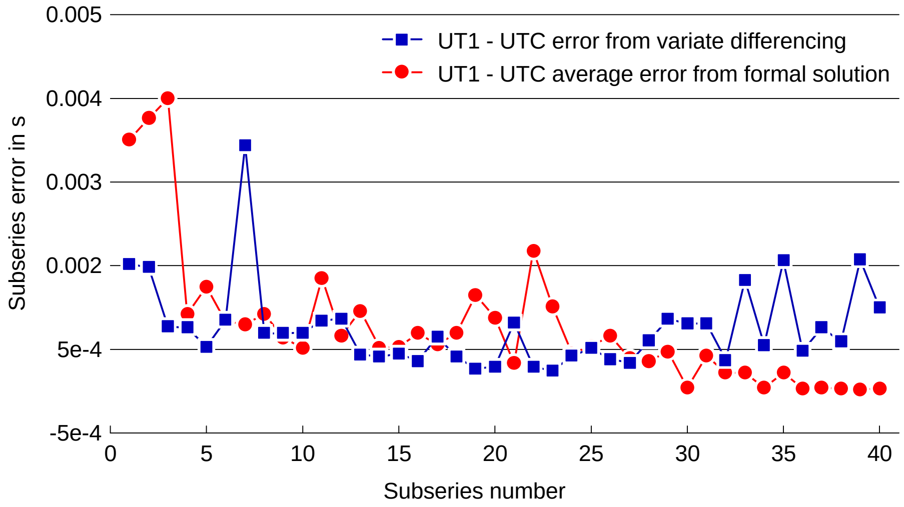

Figure 8. UT1 - UTC errors estimated by the variate differencing for each subseries and 4.5 x the average of the formal errors of each subseries.

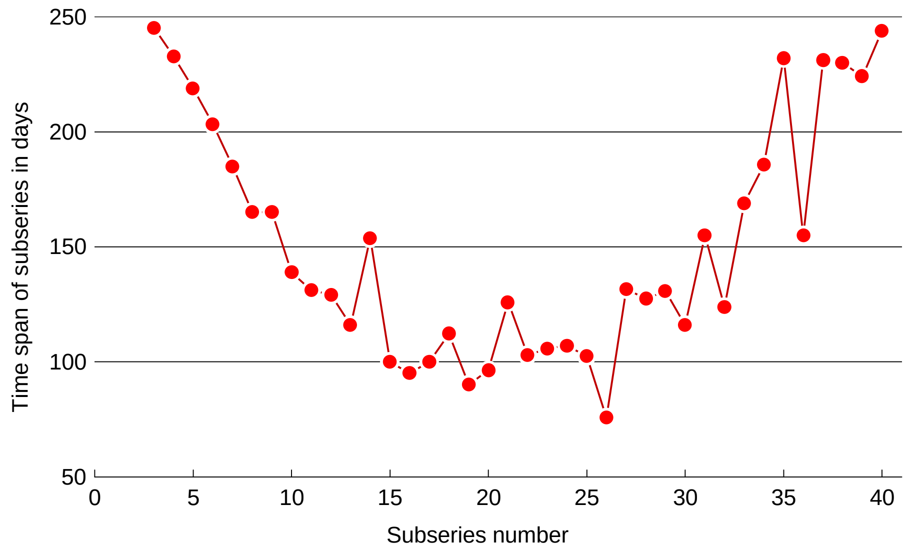

Figure 9. Although the subseries contain the same number of data points they do not span a constant period (the first series was excluded from the plot for a better view).

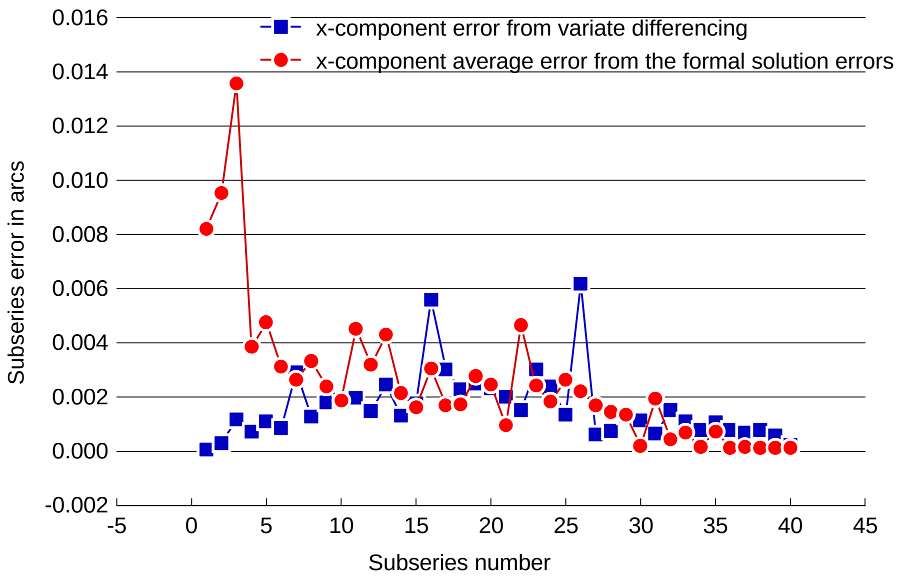

Figure 10. Polar motion x-component error estimated by the variate differencing for each sub series and the average of the formal errors of the subseries from the formal solution. Each series were normalized by the time span of series before the variate differencing.

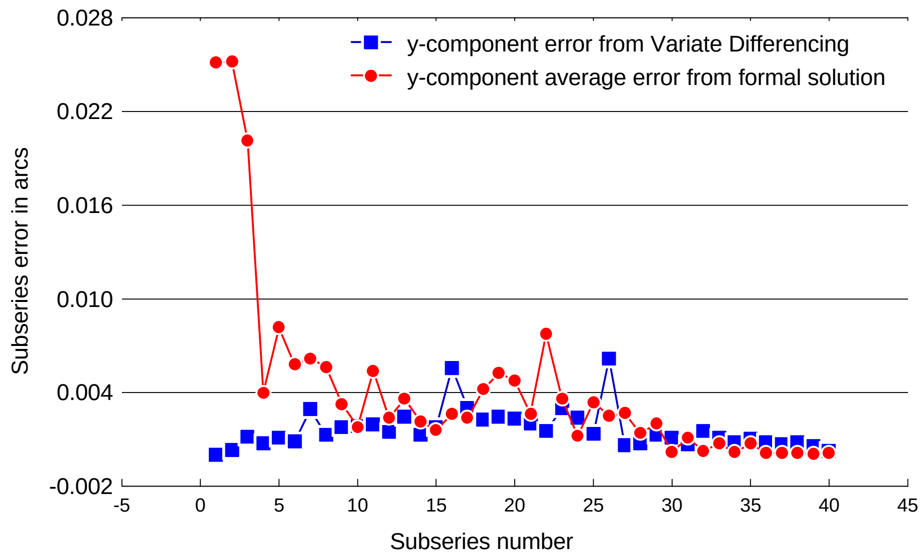

Figure 11. Polar motion y-component error estimated by the variate differencing for each subseries and the average of the formal errors of each subseries. Each series were normalized by the time span of series before the variate differencing.

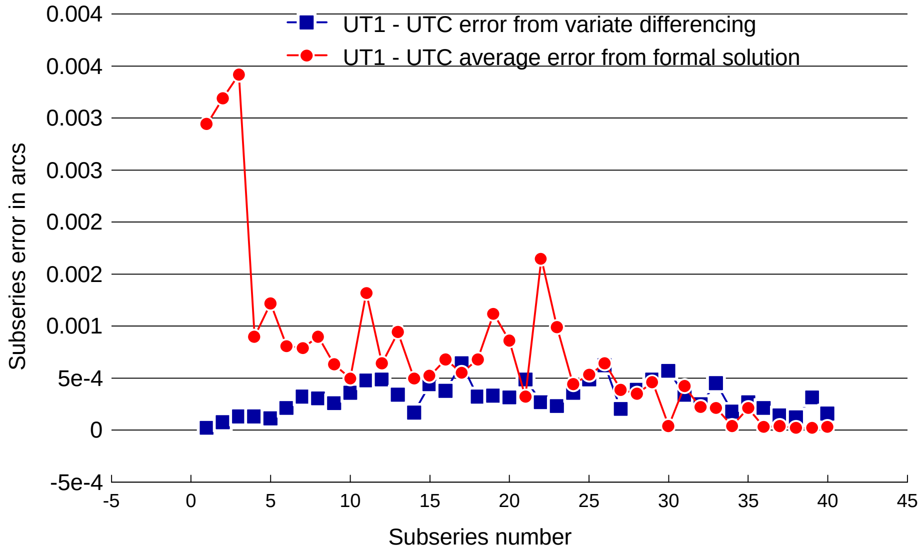

Figure 12. UT1 - UTC errors estimated by the variate differencing for each subseries and 4.5 x the average of the formal errors of each subseries. Each series were normalized by the time span of series before the variate differencing.