The Influence of the Unadjusted Satellite Magnetic Field Sources on the Recovery of the Ambient Field as Measured by the ARISTOTELES Magnetometers

Hüseyin Bâki Iz
Hughes STX Corporation
and
R.A. Langel
Goddard Space Flight Center

Summary

One of the contributions that affect the total magnetic field as measured by satellite-borne magnetometers is due to the magnetic field generated by the spacecraft itself. The accuracy of the Earth's magnetic field determination therefore depends critically on how well these effects are known in practice. Satellite magnetic sources are usually detected and their magnitudes are estimated during the ground calibration procedures. They are subsequently considered during the data processing. In this study, we examine the impact of model and data processing scenarios on the recovery of the satellite ambient field with emphasis on unadjusted satellite magnetic field sources. We considered various unmodeled satellite source magnitudes, alternative boom lengths, different satellite field locations and ambient field effects. Noise-only estimates that are due to instrument precision and mission configuration, and total error estimates that also include unadjusted satellite field effects were quantified.

1. Introduction

The determination of the ambient magnetic field by instruments on board the Aristoteles spacecraft will be complicated by magnetic fields produced by the spacecraft itself such as; magnetic fields produced by the satellite batteries, power systems, gyros, motors, relays and other magnetic material. The variable effects of switching satellite subsystems on and off, of thruster firings and of other operations that introduce magnetic disturbances are usually modeled, so these effects can be subtracted from measurements of the ambient magnetic fields in space. In any case, information about the location, magnitude and direction of all these effects is a necessity. In practice however, not only the imperfect knowledge about the satellite generated magnetic effects but also unknown stray fields and unaccounted moments induced by soft magnetic materials introduce additional uncertainties in the recovery of the ambient magnetic field.

All these effects are reduced if the magnetic field instruments are located at the end of a boom, as far from the spacecraft as possible. Yet, the length of the boom is limited and the boom deflection may introduce additional uncertainties. This problem has been examined in an earlier report (Iz and Langel, 1992).

On the other hand, however carefully calibrated on the ground, there will always be some satellite borne magnetic fields that become apparent after the satellite launch. The number of satellite magnetic sources may be quite large but their effects can be disregarded if their effect on the recovery of the ambient field is found to be small. If the satellite magnetic sources are well defined during the ground calibration procedures then it is sufficient to apply their effects as data corrections during preprocessing. They need not be estimated during the orbit calibration. For all these cases, it is necessary to quantify the influence of unaccounted satellite sources on the ambient field measurements. We focus on the following questions:

- How accurately must the satellite magnetic sources be known for them to be treated as data corrections ?
- What is the largest satellite magnetic source that can be safely ignored in the recovery of the ambient field from the magnetometer measurements?
- What is the trade-off between boom length and unadjusted satellite magnetic sources? i.e., what is the optimum boom length in the presence of unaccounted sources?
- Are there any other design parameters (besides increasing the boom length) that might reduce the influence of unmodeled satellite magnetic field sources on the ambient field calculations?

We will again consider the break down for the total error of the magnetometer instrument budget given in Table 1 which quantifies the upper limits for the effect of these uncertainties on the ambient field determination. Note that attitude error is not included.

Table 1. Magnetometer Error Budget (units are in nT).

Type	Scalar	Vector
Instrument	1.0	2.0
Spacecraft Field	1.0	1.0
rss	1.41	2.24
Position and Time	1.3	1.3
Total (rss)	1.92	2.58

In this analysis, spacecraft magnetic sources are again represented as dipoles with specified accuracy. It is assumed that major satellite magnetic sources are modeled and the data is nominally corrected for satellite sources which are not included in the overall model. The observed magnetic fields at the magnetometers due to these sources are formulated and their uncertainties are propagated, together with the effect of unadjusted magnetic field sources, into the uncertainty of the ambient field recovery at a given epoch. The scenario in which the unadjusted dipole source uncertainties are equal to the dipole magnitudes (i.e., they are in hundred percent in error) mimics the case where these particular satellite magnetic sources are unaccounted in the solution.

The influence of boom bending and twisting is assumed to be small, (Iz and Langel, 1992), and is neglected in this analysis for the sake of simplicity.

In the following sections, first the adopted satellite and instrument configuration are given. Physical and statistical modelings of the total magnetic field measurements are discussed in the subsequent sections. The error analysis results are then reported under the 'Numerical Results and Conclusion' section.

2. Instrument Configuration and Coordinate Systems

It is assumed that the magnetic field instrument package for Aristoteles consists of one fluxgate vector magnetometer and one scalar magnetometer, both located at the end of a boom. A set of star sensors at the foot of the boom together with an optical attitude transfer control system from the end of the boom relates the attitude of the instrument package to the star sensor.

Spacecraft coordinate system origin is at the far end of the spacecraft, opposite the boom. The vector and scalar magnetometers are nominally collocated on the x-axis of the spacecraft coordinate system. The vector magnetometer coordinate system is centered on the far end of the boom and is assumed to be parallel to the spacecraft system. As a result, translation of the magnetometer coordinate system in the x direction results in the spacecraft coordinate system.

3. Spacecraft Magnetic Field Model

Spacecraft magnetic fields are modeled as dipoles. The potential at a point $\mathbf{r}' = (x\ y\ z)'$ due to a magnetic dipole at $\mathbf{r}_d$ can be expressed as

$$V_d = \frac{\mathbf{m}\cdot\mathbf{u}_d}{4\pi u_d^3} = \frac{1}{4\pi u_d^3}\left[m_x(x-x_d)+m_y(y-y_d)+m_z(z-z_d)\right] \quad (1)$$

where $\mathbf{u}_d = \mathbf{r} - \mathbf{r}_d$ is the unit vector from the dipole to the point $\mathbf{r}$ (Figure 1). $\mathbf{m}' = (m_x\ m_y\ m_z)'$ is the dipole moment vector (prime denotes transpose of a vector or a matrix and bold letters are for matrices or vectors). Making use of (1), the magnetic field generated at $\mathbf{r}_d$ by this dipole moment is given, after some algebraic manipulations, by

$$\mathbf{B}_d = -\frac{\mu_0}{4\pi}\nabla V_d = -\frac{\mu_0}{4\pi u_d^3}\left[\mathbf{m} - 3\frac{\mathbf{m}\cdot\mathbf{u}_d}{u_d^2}\mathbf{u}_d\right] \quad (2)$$

where μ_0 is the permeability of free space. This model can be rearranged and expressed as

$$\mathbf{B}_{d_i} = \mathbf{A}_{d_i} . \mathbf{m}_i \tag{3}$$

where the subscript i denotes the ith dipole and

$$\mathbf{A}_{d_i} := -\frac{\mu_o}{4\pi u_{d_i}^3}\left[\mathbf{I} - 3\frac{\mathbf{u}_{\mathbf{d_i}}}{u_{d_i}^2}\mathbf{u}_{\mathbf{d_i}}^T\right] \tag{4}$$

The total magnetic field, as measured by a magnetometer at an arbitrary position on the boom, is the sum of the individual effects. It is obtained by considering (3) and (4)

$$\mathbf{B}_T^v = \mathbf{B}_A + \sum_i \mathbf{B}_{d_i} \tag{5}$$

where $\mathbf{B}_A$ and $\mathbf{B}_d$, are evaluated at their corresponding locations $\mathbf{r}$ and $\mathbf{r}_{d_i}$ The above expression is not only valid for relating the ambient field and the satellite magnetic field to the vector magnetometer components but also represents the scalar magnetometer measurements through the corresponding magnitude expression, i.e.,

$$B_T^S = \left|\mathbf{B}_A + \sum_i \mathbf{B}_{d_i}\right| \tag{6}$$

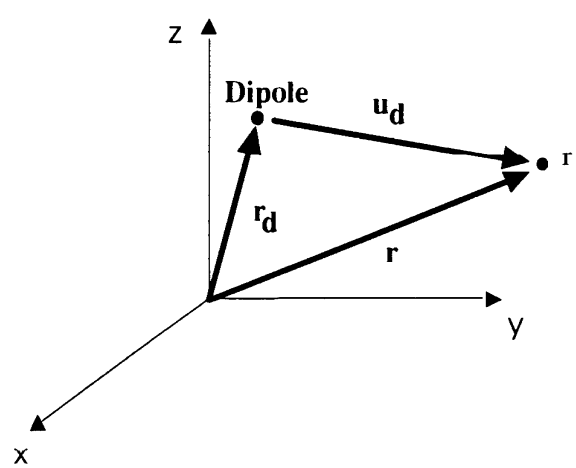

Figure 1. Dipole location in the spacecraft coordinate system.

4. Statistical Model

Making use of (3), (4), (5) and (6), the following set of equations relates the total magnetic field as observed by the scalar and vector magnetometers to the ambient field

$$\mathbf{B}_T^{v^{obs}} = \mathbf{B}_A + \sum_{i=1}^{n} \mathbf{A}_{d_i} \mathbf{m}_i + \mathbf{v} \tag{7}$$

$$B_T^{s^{obs}} = \left| \mathbf{B}_A + \sum_{i=1}^{n} \mathbf{A}_{d_i} \mathbf{m}_i \right| + s \tag{8}$$

$$\begin{aligned} \mathbf{m}_{d_1}^{p} &= \mathbf{m}_{d_1} + \mathbf{d}_1 \\ \vdots \quad & \quad \vdots \quad \vdots \\ \mathbf{m}_{d_n}^{p} &= \mathbf{m}_{d_n} + \mathbf{d}_n \end{aligned} \tag{9}$$

where $\mathbf{B}_T^{v^{obs}}$ is the 3x1 vector of vector magnetometer component measurement at a given epoch , $\mathbf{B}_A$ is the 3x1 vector of ambient field components at the same epoch, $\mathbf{v}$ is the 3x1 array of vector magnetometer component measurement disturbances with the following assumed statistical properties

$$E(\mathbf{v}) = \mathbf{0}, \qquad E(\mathbf{v}\mathbf{v}') = \sigma_v^2.\mathbf{I} \tag{10}$$

where $\mathbf{0}$ is a 3x1 zero vector. σ_v^2 is the a priori variance of unit weight of the vector magnetometer component measurements. In the case of scalar magnetometer instrument, the observations denoted by exponent indices "s," are scalar quantities with the following assumed statistical properties for their errors,

$$E(s) = 0, \qquad E(s^2) = \sigma_s^2 \tag{11}$$

Stochastic noise vector **v** and scalar noise s are further assumed to be uncorrelated, i.e.,

$$E(\mathbf{v}s) = \mathbf{0} \tag{12}$$

$\mathbf{m}_{d_i}^{p}$ is the 3x1 vector of spacecraft field dipole moments magnitudes and available from laboratory calibration procedures. The 3x1 corresponding dipole error vector $\mathbf{d}_i$ has the following assumed distributional properties

$$E(\mathbf{d}_i) = \mathbf{0}, \qquad E(\mathbf{d}_i\mathbf{d}_i') = \sigma_{d_i}^2.\mathbf{I} \tag{13}$$

This random error is also assumed to be independent of the vector and scalar magnetometer measurements v and s. For n dipoles, (13) introduces 3n x1 vector of dipole moment components with 3n x 3n error covariance matrix. Note that (7) and (9) are linear in terms of the dipole moments and ambient field components whereas (8) is not. Expressing (8) in Taylor series and retaining only the linear terms,(7), (8) and (9), can be collected into a single matrix expression as follows

$$\Delta\mathbf{y} = \mathbf{A}\Delta\mathbf{x} + \mathbf{u} \tag{14}$$

where

$$\mathbf{A} := \begin{bmatrix} \mathbf{A}_d \\ \mathbf{x}'\mathbf{A}_d'\,\mathbf{A}_d\left(\mathbf{x}'\mathbf{A}_d'\,\mathbf{A}_d\mathbf{x}\right)^{-1} \\ \mathbf{I} \quad \mathbf{0} \end{bmatrix}_{\mathbf{x}=\mathbf{x}_0} \tag{15}$$

$$\mathbf{u}' := \begin{bmatrix} \mathbf{v} & s & \mathbf{d} \end{bmatrix}' \tag{16}$$

$$\Sigma_u := \begin{bmatrix} \sigma_v^2 . I & 0 & 0 \\ 0 & \sigma_s^2 & 0 \\ 0 & 0 & \sigma_{\mathbf{d}}^2 . I \end{bmatrix} \tag{17}$$

$$\Delta\mathbf{y} := \mathbf{y}_{obs} - \mathbf{y}_0' \tag{18}$$

$$\mathbf{y}_{obs}' := \begin{bmatrix} \mathbf{B}_T^{v\,obs} & B_T^{s\,obs} & \mathbf{m}^p \end{bmatrix}' \tag{19}$$

$$\mathbf{y}_0' := \begin{bmatrix} \mathbf{B}_T^v & B_T^s & \mathbf{m}^p \end{bmatrix}'_{\mathbf{x}=\mathbf{x}_0} \tag{20}$$

$$\Delta\mathbf{x} := \mathbf{x} - \mathbf{x}_0 \tag{21}$$

$$\mathbf{A}_d := \begin{bmatrix} \mathbf{A}_{\mathbf{d}_1} & \ldots & \mathbf{A}_{\mathbf{d}_n} & \mathbf{I} \end{bmatrix} \tag{22}$$

$$\mathbf{x}' := \begin{bmatrix} \mathbf{m}_1' & \ldots & \mathbf{m}_n' & \mathbf{B}_A' \end{bmatrix} \tag{23}$$

$$\mathbf{x}_0' := \begin{bmatrix} \mathbf{m}_1' & \ldots & \mathbf{m}_n' & \mathbf{B}_A' \end{bmatrix}_0 \tag{24}$$

The zero indice indicates that the corresponding quantity is evaluated at the adopted nominal values of the parameters. $\mathbf{x}$ is the $(3n + 3) \times 1$ vector of parameters (true values) consists of 3n dipole components that are known from the calibration procedures and 3 ambient field components, If the above model is a realistic representation of the satellite magnetic sources that affect the magnetometer measurements, it can be solved using the well known least-squares procedures and the effect of the error sources on the ambient field can be computed accordingly using the following relationships,

$$\hat{\mathbf{x}}_1 = \mathbf{x}_1^0 + (\mathbf{A}_1'\Sigma_u^{-1}\mathbf{A}_1)^{-1}\,\mathbf{A}_1'\Sigma_u^{-1}\Delta\mathbf{y} \tag{25}$$

$$\Sigma_{\hat{\mathbf{x}}_1} = (\mathbf{A}'\Sigma_u^{-1}\mathbf{A})^{-1} \tag{26}$$

where circumflex denotes the corresponding parameter is an estimate.

Alternatively, some of the model parameters, in this case satellite magnetic sources, can be left out (i.e., not adjusted) if their influence on the magnetometer measurements are found to be negligible. To asses the influence of various satellite sources, consider the following representation in which now the model parameters are represented in two groups. The first group of parameters, which will be adjusted, includes ambient field components and 3n dipole components. The second group of parameters consists of 3m dipole moment components where m is the number of satellite sources. This second group of parameters will not be adjusted in the solution, but the magnetometer measurements will be corrected for some a priori values of these parameters,

$$\Delta\mathbf{y} := \mathbf{A}_1\Delta\mathbf{x}_1 + \mathbf{A}_2\Delta\mathbf{x}_2 + \mathbf{u} \tag{27}$$

where $\mathbf{A}_1\Delta\mathbf{x}_1$ is defined by the corresponding expressions in (15)-(24), $\mathbf{A}_2\Delta\mathbf{x}_2$ represents the unadjusted satellite field effects on the magnetic field measurements. $\Delta\mathbf{x}_2$ is the 3m x 1 vector of unadjusted m satellite dipoles where

$$\mathbf{A}_2 := \begin{bmatrix} \mathbf{A}_d^2 \\ \left(\mathbf{x}_2'\mathbf{A}_d'^2\mathbf{A}_d^2\mathbf{x}_2'\right)^{-1}\mathbf{A}_d'^2\mathbf{A}_d^2\mathbf{x}_2 \end{bmatrix} \tag{28}$$

$$\mathbf{A}_d^2 := \begin{bmatrix} \mathbf{A}_{d_{n+1}} & \cdots & \mathbf{A}_{d_{n+m}} \end{bmatrix} \tag{29}$$

In obtaining a least squares solution to (27) with respect to ambient field components, we will omit the satellite dipole effects represented by the second term in (27) but make use of the following form,

$$\Delta\mathbf{y} = \mathbf{A}_1\Delta\mathbf{x}_1 + \mathbf{u} \tag{30}$$

Taking into consideration (27), the error committed of using an incomplete model given (30) can be assessed using the well-known definition of the Mean Square Error matrix

$$\mathbf{MSE}(\hat{\mathbf{x}}_1) := \mathbf{E}(\mathbf{x}_1 - \hat{\mathbf{x}}_1)(\mathbf{x}_1 - \hat{\mathbf{x}}_1)' \tag{31}$$

which reduces after some lengthy matrix manipulations, considering (27) as the true model, to

$$\mathbf{MSE}(\hat{\mathbf{x}}_1) = \mathbf{N}_1^{-1} + \mathbf{N}_1^{-1}\mathbf{A}_1'\Sigma_u^{-1}\mathbf{A}_2\Sigma_{x_2}\mathbf{A}_2'\Sigma_u^{-1}\mathbf{A}_1\mathbf{N}_1^{-1} \tag{32}$$

$$\mathbf{N}_1 := (\mathbf{A}_1'\Sigma_u^{-1}\mathbf{A}_1)^{-1} \qquad \Sigma_{x_2} := E(\Delta\mathbf{x}\Delta\mathbf{x}^T) = \sigma_{x_2}^2.\mathbf{I} \tag{33}$$

In the above expression, $\sigma_{x_2}^2$ is the uncertainty attributed to the unadjusted dipoles. Note that if the uncertainty of the unadjusted dipole is zero than the second term in (32) drops and the remaining expression is nothing but the covariance matrix of the model described by (14) - (26). Therefore the second term in (32) is the increase in the total error (MSE) due to nominally corrected but unadjusted satellite dipole sources. As expected, if these sources are well known —they possess smaller uncertainties—then they need not to be adjusted but introduced as a correction to the data. Larger uncertainties imply that they are lesser and lesser known and they are more influential if the corresponding signals are also large. A signal to noise ratio of one for instance can be used to evaluate the full impact of these sources if they are completely omitted from the solution, i.e., data is neither preprocessed or included in the model adjustment.

The error expression given by (32) can now be systematically evaluated for various design parameters such as dipole strengths and orientations, dipole locations, boom lengths, magnetometer precisions, and ambient field components.

5. Numerical Results and Conclusion

Satellite magnetotorquers are the major source of satellite magnetic fields, Vittone and Maggi (1992). Their magnetic data characteristics; magnitudes, their uncertainties as determined by the calibration procedures, are given in Table 2. Their location vectors are expressed in the satellite coordinate system.

Table 2. Magnetic data characteristics (x, y, z) coordinates are in meters. Magnetic data components (m_x, m_y, m_z) are in Am^2. Data is in satellite coordinate system as defined in Vittone and Maggi (1992).

x	y	z	m_x	m_y	m_z	σ	Description
0.992	-0.395	0.648	20.0	0.0	0.0	2	Torquer 1
0.991	-0.019	0.648	0.0	20.0	0.0	2	Torquer 2

Although, several other satellite sources exist, some of them are at least one order of magnitude less than the magnetotorquer magnitudes. Should these sources be included in the model and adjusted together with these two sources, or should the data only be corrected for their effects? Figure 2 reflects a simple scenario to illustrate this problem.

Table 3. Fixed and variable design parameters.

Parameter	Range of values
Boom lengths (m)	3, 4, 5
Unadjusted dipole values/uncertainties (Am^2)	0.1, 1, 10
Vector and scalar magnetometer precision (nT)	2, 1
Nominal ambient fields (nT)	(23000, 0, -23000)
	(0, 0 , 23000)
	(-23000, 0, 0)
Unadjusted satellite field locations (m)	(0.5, 0.0, 0.0)
	(0.0, 0.5, 0.0)
	(0.0, 0.0, 0.5)

In this case, magnetometers are assumed to be located on a boom of 3 m, as far from the torquers as possible. The satellite coordinate system is at the far side of the spacecraft which increases the instrument-to-source distance in the x direction about 5 m. A third source of magnitude 1 Am^2, left unadjusted, is located at (0.5, 0, 0) in the satellite coordinate system. This unadjusted source increases the uncertainty of the ambient field 3.8 percent in the x component due to the uncertainty of the instrument and adjusted torquer effects. At this point, even the noise only portion of the solution, i.e., no unmodeled sources, is barely within the mission requirement level.

Table 3 is a summary of the design parameters we will consider for various scenarios. Consider now first the case where all the above parameters defined for figure 2 remain the same but the instruments are located further away from the unadjusted source (4, and 5 m) deploying the magnetometers at the end of a longer boom.

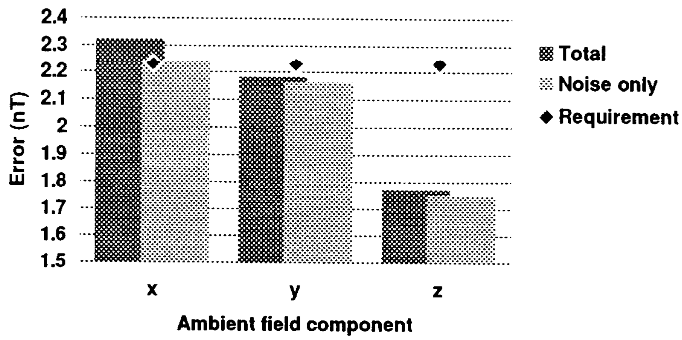

Figure 2. The impact of an unadjusted dipole source of magnitude 1 Am^2 in the x direction of the satellite coordinate system, to the computed ambient field components. Boom length is 3 m (on a 5 m satellite length). Adjusted satellite sources with components 20 Am^2 in the x and y directions of the satellite coordinate system (Table 2). Vector and scalar magnetometer precisions are 2 nT and 1 nT respectively.

The influence, thereby the uncertainty of the unadjusted satellite magnetic source on the ambient field recovery, reduces, as expected, as the instruments are located further away from the unadjusted source. This reduction occurs more rapidly on the x-component of the ambient field uncertainty because the unadjusted source and the instruments are all located on the x-axis of the satellite coordinate system (i.e., the influence is proportional to the source-to-instrument distance which varies more rapidly in this configuration in the x direction).

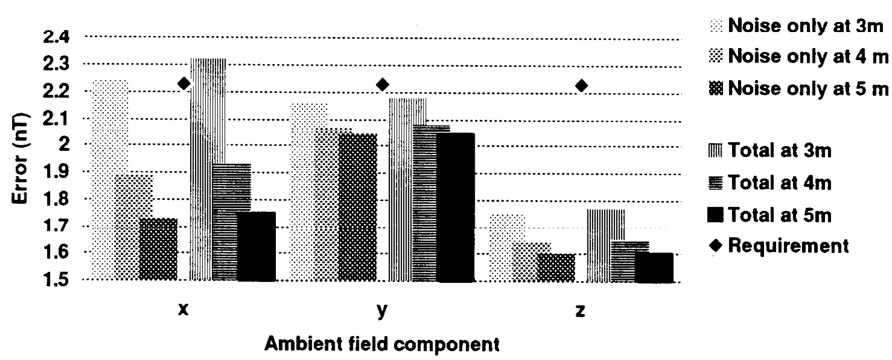

Figure 3. The impact of an unadjusted dipole source known within 1 nT accuracy at 3, 4, 5 m boom lengths.

In Figure 4 the influence of satellite source magnitudes of 0.1, 1, 10 Am^2 is displayed. The instruments are assumed to be located at the end of a boom of 4m length. The other configuration parameters remain the same as in the previous scenario. The contribution of the unadjusted source again increases with increasing source magnitude. But they are more pronounced at 10 Am^2 as compared to a source of 1 Am^2. Both x and y component total errors exceed the mission budget constraint. Nevertheless a source of this magnitude is easily detected and the data are adjusted (corrected). If there is a mismodeled portion, say, on the order of 10 percent of the source magnitude, then the results displayed in Figure 4 for 0.1 Am^2 satellite source magnitude can be used to assess the mismodeling effect.

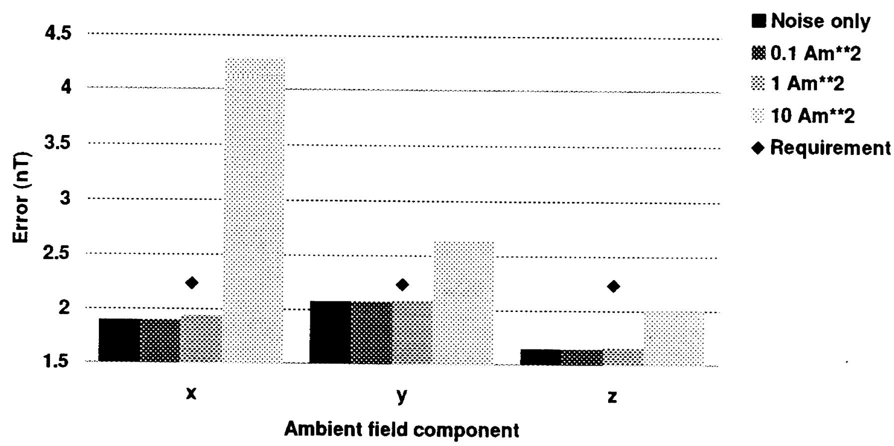

Figure 4. The impact of satellite sources of magnitude 0.1, 1, and 10 mA^2 on the adjusted ambient field components at 4 m boom length.

The error estimates of the ambient field components due to a source located at various field locations (Table 2) are given in Figure 5. Again, because the impact of the unadjusted source on the ambient field components is proportional to the distance between the instruments and the source, the effect is most influential at the x direction where the distance between them is closest. For smaller unadjusted source magnitudes the variation of the location effect on the ambient field is negligibly small.

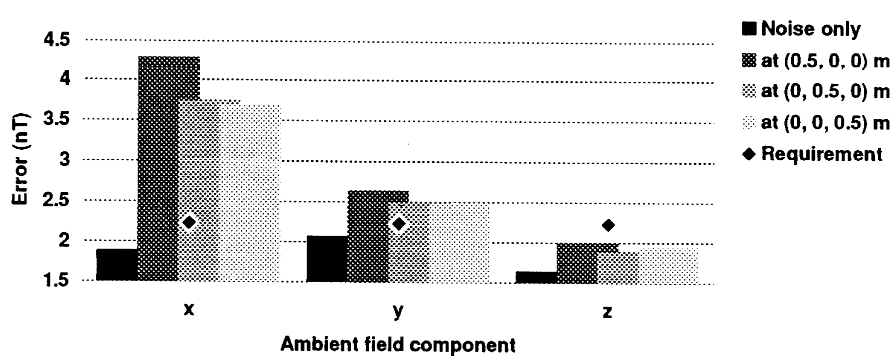

Figure 5. The variation in error as a result of an unadjusted satellite source at different locations. Boom length is 4 m, unadjusted source magnitude is 0.1 mA^2.

The error on the ambient field recovery as a result of unadjusted field may also be compounded or diminished by the magnitude of the ambient field observed. The results displayed in Figure 6 indicate that these effects do indeed exist and need to be accounted in cases where more accuracy in the recovery is needed.

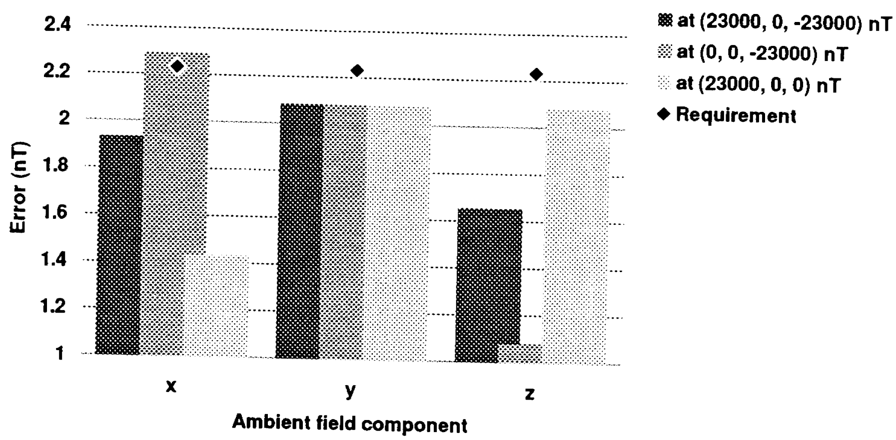

Figure 6. The influence of field magnitude is compounded or diminished with the unadjusted satellite field effect. Satellite source magnitude is 1 mA^2 at 4 m boom length.

References

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