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THE STOCHASTIC ESTIMATION OF SATELLITE CLOCK CORRECTION INFORMATION IN WADGPS

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ABSTRACT Using autocorrelation information of the pseudorange errors generated by selective availability (SA) frequency dithering, we have constructed a simple first order stochastic model for SA effects. This model has been used in a Kalman filter to account for the stochastic behavior of SA dithering in estimating satellite clock information in wide area differential GPS. We have obtained fifteen percent improvement in the user positioning using the correlation information on the satellite clock information in a Kalman filter, when comparing the results obtained using a regular least square estimation.

1 Introduction

Conventional DGPS is limited by the range over which the differential corrections are valid due to the rapid decorrelation of the error sources with increasing distance from the reference station to user. In wide area differential GPS (WADGPS) error sources in GPS measurements are modeled separately, on the basis of a limited number of reference staitons, to overcome this drawback. The main error sources are regarded as broadcast ephemeris error, atmospheric refraction and satellite clock offset and SA dithering (Ashkenazi *et al.*, 1993; Chao, 1996).

Usually, a near real-time precise satellite ephemeris and a regional atmospheric correction model are predicted using data collected at the reference stations. The data are transmitted to all users to replace the SA-downgraded broadcast ephemeris and the Klobuchar ionospheric model is included in the GPS navigation message. Satellite clock correction information, a combination of satellite clock offset and SA dithering, is then estimated using predicted ephemeris, ionospheric model and pseudorange observations recorded at all reference stations (Kee *et al.*, 1991)

In most of current WADGPS software, least square method is used to estimate the satellite clock correction epoch by epoch. Because the predicted satellite clock correction model with accuracy of better than a few meters is given in satellite navigation message, the main error of satellite clock correction information can be attributed to SA dithering. The SA effect on measurements is the integration of the SA frequency dithering, which is about 1 to 2 Hz, i. e., 0.2 to 0.5 m/s. Consequently, the observations become serially correlated because of the smoothing effect of SA through integration. Their realizations through the estimation process should therefore account for the autocorrelation. This additional information will improve the estimation of SA parameters thereby improves WADGPS results.

We have obtained the SA data from DMA for the field data used in the analysis. After the analysis of

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the stochastic behavior of SA dithering using true SA data, stochastic model is introduced into parameter estimation using a Kalman filter. The field data are then used to demonstrate the improvement of the user positioning accuracy by considering the stochastic property of SA dithering.

2 Stochastic behavior of SA dithering

Fig. 1 shows the SA clock dithering effect on the pseudorange of PRN03. The range error caused by SA dithering has a maximum magnitude of 70m. Harmonic analysis does not reveal any significant periodic terms in the SA dithering time series.

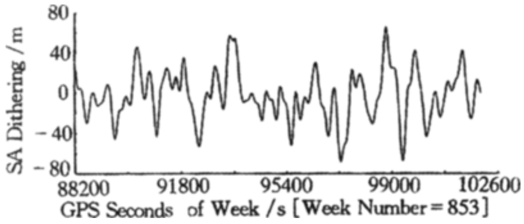

Fig. 1 SA effect on pseudorange measurements

The autocorrelation coefficient of a time series is defined as follows

$$\rho(\tau) = \frac{\sum_{i=1}^{n-\tau} SA_{i+\tau} \times SA_i}{\left[\sum_{i=1}^{n-\tau} SA_{i+\tau}^2 \sum_{i=1}^{n-\tau} SA_i^2\right]^{\frac{1}{2}}} \qquad (1)$$

where τ denotes the time lag. Fig. 2 shows the correlogram of SA data that indicate strong correlation in time for nearly spaced data. This effect can be represented as a first order random process:

$$SA_{i+1} = \rho(\tau) SA_i + \varepsilon_{i+1} \qquad (2)$$

where, ε_i is a random noise with $E(\varepsilon_i) = 0$. Although the autoregressive behavior of the SA data is represented better by a second order Gauss-Markov process for the existing SA data, there is no guarantee that DoD will maintain this property when the SA dithering data is generated. On the other hand , a first order autoregressive effect will always be present because of the SA data integration. Moreover, for shorter sampling intervals, such as 1 second which is typical in real-time differential GPS positioning, the autocorrelation of the serial data will be much higher and the first order effect will dominate the results. Fig. 2 shows the estimated correlogram of the SA dithering for lags up to three hours.

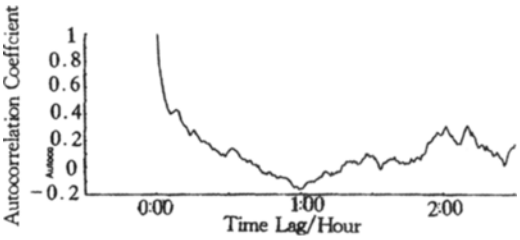

Fig. 2 Estimated correlogram of SA dithering for lags up to three hours

Using the postulated first-order autoregressive process, the variance of the random process $\sigma^2(\tau)$ can be obtained from the estimated noise $\hat{\varepsilon}_i$ using Eq. (2) and estimated autocorrelation coefficient:

$$\sigma^2(\tau) = \sum_{i=1}^{n} \varepsilon_i^2 (n-1) \qquad (3)$$

The autocorrelation coefficient calculated using true SA data provided by DMA is about 0.99 for all satellites with one-second data sampling rate, and the corresponding variance is between 0.09 m^2 to 0.36 m^2. For data sampling rate at k seconds intervals, according to the autoregressive model given by Eq. (2), the corresponding correlation coefficient and the variance are $\rho(\tau)^k$ and $k\sigma^2(\tau)$ respectively.

We will use the estimated first-order autoregressive model, as a state equation in a Kalman filter in the estimation of the satellite clock correction.

3 Estimation of satellite clock correction

The statistical model that relates observed pseudoranges to the satellite clock correction , SA dithering and receiver clock offset can be written as

$$\Delta l_{ij} = \Delta t_{rj} - \Delta t_{si} + v_{ij} \qquad (4)$$

where, Δt_{si} is satellite clock correction parameter to be estimated . It includes the satellite clock offset and SA dithering; Δt_{rj} is receiver clock offset; Δl_{ij} is the correction to the observed pseudorange (i. e. observed pseudorange minus computed pseudorange). We assume that pseudorange observations are correlated with the elevation of the satellite. P_{ij} is the corresponding weight that is assigned to individual observation according to the elevation of the satellite. i and j indicate receiver and satellite number.

Current practice in most WADGPS applications is to estimate satellite correction prameters using a least squre estimator. Due to the singularity of the normal equation, a reference clock is selected for each epoch and relative clock correction to the reference clock is estimated.

It is well known that a prior information about the parameters always improves the corresponding estimates. Improvement is possible even when the prior information is wrong under certain circumstances (Iz, 1987) . Marked improvements can be obtained, that the prior uncertain information is smaller then the noise of the observations in cases that there is little information supplied by the obeservations. In WADGPS , the variance of the pseudorange observations is about 0.36 m^2 without AS and 1 m^2 to 4 m^2 with AS , which is much larger than the estimated variant of SA dithering (0.09 ~0.36m^2). We have therefore chosen to introduce additional information in the estimation process about the clock correction via a Kalman filter.

4 Discussion of numerical results and conclusion

The satellite clock offset can be removed using clock information in navigation message before the estimation, hence the data contain only the effect of the SA dithering. Prior informatin about the satellite clock parameter (estimated autoregressive model) is introduced using the following state equations for the SA dithering and receiver clock behavior for observation data collected at 1 Hz,

$$\begin{aligned} \Delta t_{s,k+1} &= \rho(\tau)\Delta t_{s,k} + w_{s,k+1} \\ E(w_{s,k+1}) &= 0, D(w_{s,k+1}) = 0.25\mathrm{m}^2/\mathrm{s} \quad (5) \\ E(\Delta t_{s,0}) &= 0, D(\Delta t_{s,0}) = 2500\mathrm{m}^2 \end{aligned}$$

State equation for receiver clock parameter is:

$$\begin{aligned} \Delta t_{r,k+1} &= \Delta t_{r,k} + w_{r,k+1} \\ E(w_{r,k+1}) &= 0, D(w_{r,k+1}) = 9\mathrm{m}^2/\mathrm{s} \quad (6) \\ E(\Delta t_{r,0}) &= 0, D(\Delta t_{r,0}) = 10\,000\mathrm{m}^2 \end{aligned}$$

To validate the usefulness of introducing additional information via stochastic estimation of satellite correction information in WADGPS, we have modified the software on the WADGPS clock information estimation provided by WTUSM (Wuhan Technical University of Surveying and Mapping, Liu *et al.*, 1997).

The modified software predicts a near real-time precise ephemeris using data collect at the reference stations. The ionospheric refraction is expressed as a quadrate polynomial function of latitude and local sidereal time of the intersection point of the signal path with the ionospheric layer . Using the predicted ephemeris and ionospheric model, satellite clock correction are estimated by a least square estimator or a Kalman filter.

In data processing, range/phase smoothed range data are used. Observations collected in Beijing using Rogue 8 000 receivers at 30s interval at four reference stations were used to predict an ionospheric model. Satellite clock corrections were estimated using observations collected in one-second sampling intervals at two other stations and using DMA precise ephemeris and the predicted ionospheric model.

Fig. 3 shows the difference between the estimated SA dithering and the true SA dithering of PRN03. The estimated SA dithering is obtained by subtracting clock offset provided by DMA precise ephemeris from estimated satellite clock correction. The difference is an effective indicator of the accuracy of the estimated parameters. To compare the stochastic estimation with the conventional least square estimation clock correction informatin was also estimated using a least-square estimator.

We have used both results (satellite correction information) to calculate single point positioning epoch by epoch at a known user location. The deviations of the calculated position from the known position of the user indicate the impact of the two different solutions.

We use the field data collected at two reference stations at one-second interval to estimate the satellite clock information. The estimated pseudorange noise was about 2m , significantly larger than the random error of SA dithering. Fig. 4 shows the user positioning error using estimated clock correction information, Kalman filter and least square estimator respectively. The correlation coefficient and the variance are set to the estimated values using true SA data, namely ,0.99 and 0.62 m^2 respectively. Each point represents an average position error of four epochs with one-second interval. The 2m means that position error using clock correction information calculated by Kalman filter offers 15% improvement when comparing the mean position error, 2.3m, with clock information using least square estimation.

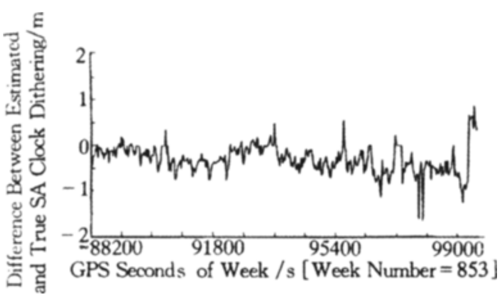

Fig. 3 The difference of estimated SA dithering and true SA

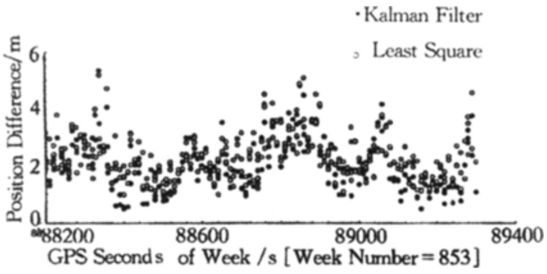

Fig. 4 Positioning error using Kalman filter with autoregressive model and using least square estimation

We recognized that if the variance of the random error of SA dithering is set too large in the stochastic model then the state equation have no effect as expected. Too small variances caused systematic errors in the estimated satellite clock information because the constraint imposed by the additional information (first order autoregressive model) is not totally correct. If the variance of the random error of the SA dithering is chosen between 0.6m^2 to 1m^2 at one-second sampling rate and the autocorrelation coefficient is set to be larger than 0.98, then the Kalman filter gives the same improvement of the least squares solution. If correlation coefficient is smaller than 0.95, it would give worse results due to the incorrect stochastic constraint for SA dithering.

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