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VLBI Baseline Rates from Baseline Measurements of Collocated Antennas using Composite Models

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Abstract

At a given site, occasionally a new VLBI antenna has replaced an older one for operational use. Large-scale VLBI solutions account for these situations by assuming the motion of the old and new antenna is the same. We alternatively used composite models in which the common baseline rates are represented directly and estimated from collocated baseline measurements. Because the baseline lengths are invariant under rotations and translations, the baseline models are less prone to the common systematic effects and consequently simpler than those of large-scale solutions. We investigated the compatibility of the baseline measurements from different nearby antennas and compared the composite model solutions with the single baseline solutions for baseline measurements from the a) ALGOPARK to NRAO85-3 and NRAO20, b) GILCREEK to NRAO85-3 and NRAO20, and c) HARTRAO to KAUAI and KOKEE antennas. We have found data that contribute significantly to the estimated rates and calculate solutions that are more robust. We also evaluate the gain in efficiency in estimating baseline rates because of the increased number of observations in the composite models.

1. Baseline Measurements

The majority of VLBI baseline measurements are generated on a global basis through the observing programs of NASA's Goddard Space Flight Center, in close cooperation with the U.S. Naval Observatory's Earth Orientation Department, and other national and international participants. The baseline data used in this study is from the NASA Goddard Space Flight Center's VLBI terrestrial reference frame solution number 1102g, 1998 August. At a given site, occasionally a new VLBI antenna has replaced an older one for operational use. Large-scale VLBI solutions account for these situations by assuming the motion of the old and new antenna is the same. In this study, we alternatively use composite models in which the common baseline rates are represented directly and estimated from collocated baseline measurements. Because the baseline lengths are invariant under rotations and translations, the baseline models are less prone to the common systematic effects and consequently simpler than those of large-scale solutions. We investigate the compatibility of the baseline measurements from different nearby antennas and compared the composite model solutions with the single baseline solutions for baseline measurements from the a) ALGOPARK to NRAO85-3 and NRAO20, b) GILCREEK to NRAO85-3 and NRAO20, and c) HARTRAO to KAUAI and KOKEE antennas. Although most of the time the measurements are the products of weekly session (network) solutions, the three baselines with collocated antennas have varying numbers of measurements. Figure 1 shows the baseline measurements from ALGOPARK to NRAO85-3 and NRAO20 for a total of 117. The NRAO85-3 antenna was replaced by a new one within one km that became operational in October 1995 (54 measurements). The GILCREEK to NRAO85-3 and NRAO20 baselines consist of 561 measurements; 116 of them are from the GILCREEK to the new NRAO20 antenna (figure 2). The baseline measurements are

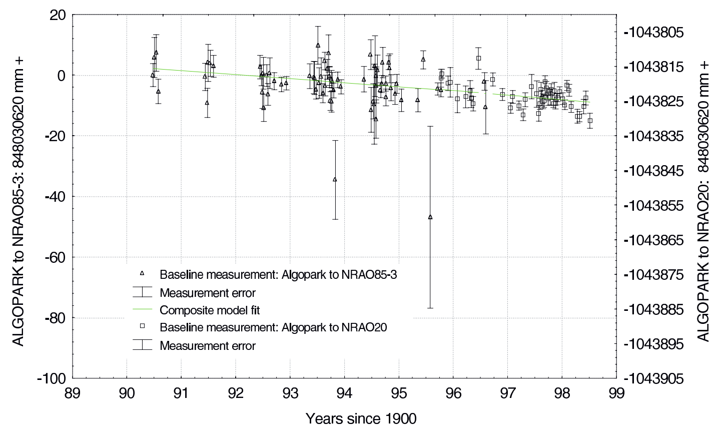

Figure 1. Baseline measurements: ALGOPARK to NRAO85-3 and NRAO20. The left Y-axis is for the ALGOPARK – NRAO85-3 baseline measurements. The offset between the two Y-axes is the difference in the intercept parameters.

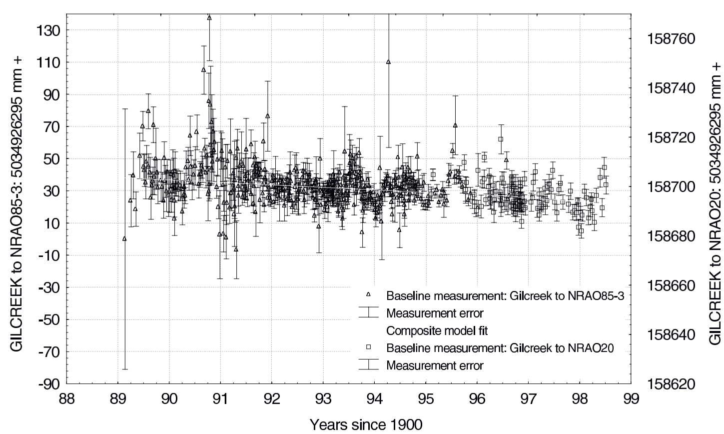

Figure 2. Baseline measurements: GILCREEK to NRAO85-3 and NRAO20. The left Y-axis is for the GILCREE – NRAO85-3 baseline measurements. The offset between the two Y-axes is the difference in the intercept parameters.

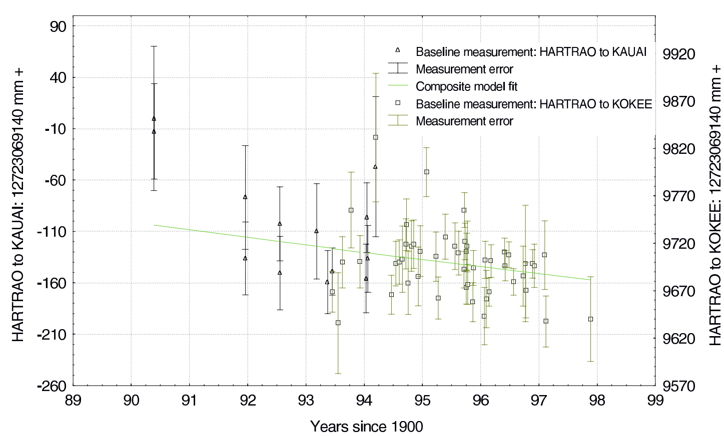

Figure 3. Baseline measurements: HARTRAO to KAUAI and KOKEE. The left Y-axis is for the HARTRAO – KAUAI baseline measurements. The offset between the two Y-axes is the difference in the intercept parameters.

more complete than the previous case but some of the early results are noisier. There are only 61 measurements from the HARTRAO to KOKEE and KAUAI baselines (figure 3). Only 13 data points belong to the HARTRAO to KAUAI baseline.

2. Single Baseline Model

The typical approach for estimating baseline rates is as follows,

$$y_{1_i} = a + bt_i + u_{1_i} \qquad i = 1, ..., m \tag{1}$$

In the above expression, y_{1_i} denotes the m baseline observations, a and b are the intercept and the slope parameters, t_i is the epoch of the observation expressed in Julian years that are shifted by an appropriate number of years in order to work with fewer digits for well conditioned solutions. The subscript 1 indicates that the single baseline model represents the data. The above expression in matrix notation reads as,

$$\boldsymbol{y}_1 = \boldsymbol{A}_1\boldsymbol{x}_1 + \boldsymbol{u}_1 \tag{2}$$

The $m \times 1$ vector of disturbances, $\boldsymbol{u}_1$, has the following assumed statistical properties,

$$E(\boldsymbol{u}_1) = \boldsymbol{0}, E(\boldsymbol{u}_1\boldsymbol{u}_1) = diag(\sigma^2_{u_{1_i}}) = \sigma^2_1 \cdot \boldsymbol{W}_1 \tag{3}$$

where σ^2_1 is the *a priori* variance of unit weight which is assumed to be unity. This value is subsequently replaced by the estimated *a posteriori* variance of unit weight.

A weighted least square solution to equation (2) is well-known and is given by,

$$\hat{\boldsymbol{x}}_1 = (\boldsymbol{A}_1\boldsymbol{W}_1^{-1}\boldsymbol{A}_1)^{-1}\boldsymbol{A}_1\boldsymbol{W}_1^{-1}\boldsymbol{y}_1 \tag{4}$$

where $\hat{\boldsymbol{x}}_1$ is the 2×1 vector of the least-square estimate of the unknown parameter vector $\boldsymbol{x}_1$ which consists of the intercept and the slope.

The following expression gives the *a posteriori* variance of unit weight $\hat{\sigma}^2_1$ of the observations,

$$\hat{\sigma}^2_1 = \frac{(\boldsymbol{y}_1 - \boldsymbol{A}_1\hat{\boldsymbol{x}}_1)\boldsymbol{W}_1^{-1}(\boldsymbol{y}_1 - \boldsymbol{A}_1\hat{\boldsymbol{x}}_1)}{m-2} \tag{5}$$

3. Composite Model

The following composite model includes two intercepts to accommodate the collocated antennas [1]

$$y_{2_j} = \alpha a_1 + \beta a_2 + bt_j + u_{2_j} \qquad j = 1, ..., n \tag{6}$$

$\alpha = 1$, $\beta = 0$ when the measurements refer to the initial baseline, and $\alpha = 0$, $\beta = 1$ when the measurements refer to the baseline with collocated antenna. The subscript in y is to denote that the data is represented by the composite model. We express the observation of the new model in matrix notation as

$$\boldsymbol{y}_2 = \boldsymbol{A}_2\boldsymbol{x}_2 + \boldsymbol{u}_2 \tag{7}$$

[1] Or alternatively, one intercept and a parameter to represent the offset between collocated antennas.

The $n \times 1$ vector of disturbances, $\boldsymbol{u}_2$, has the following assumed statistical properties,

$$E(\boldsymbol{u}_2) = \boldsymbol{0}, E(\boldsymbol{u}_2\boldsymbol{u}_2) = diag(\sigma^2_{u_{2_j}}) = \sigma^2_2 \cdot \boldsymbol{W}_2 \tag{8}$$

where the unknown parameter vector $\boldsymbol{x}_2$ includes two intercept parameters and a slope parameter due to the plate motion common to all observations. A weighted least square solution to this new model is given by,

$$\hat{\boldsymbol{x}}_2 = (\boldsymbol{A}_2\boldsymbol{W}_2^{-1}\boldsymbol{A}_2)^{-1}\boldsymbol{A}_2\boldsymbol{W}_2^{-1}\boldsymbol{y}_2 \tag{9}$$
$$\Sigma_{\hat{\boldsymbol{x}}_2} = \hat{\sigma}^2_2(\boldsymbol{A}_2\boldsymbol{W}_2^{-1}\boldsymbol{A}_2)^{-1}$$

The *a posteriori* variance of unit weight of this solution is

$$\hat{\sigma}^2_2 = \frac{(\boldsymbol{y}_2 - \boldsymbol{A}_2\hat{\boldsymbol{x}}_2)\boldsymbol{W}_2^{-1}(\boldsymbol{y}_2 - \boldsymbol{A}_2\hat{\boldsymbol{x}}_2)}{n-3} \tag{10}$$

4. Are the Data from Two Different Baselines Compatible?

The *a posteriori* variances of unit weight of the simple and composite models are statistically *dependent*. However, the estimated variance of the *added* observations given by

$$\hat{\sigma}^2_{21} = \frac{f_2\hat{\sigma}^2_2 - f_1\hat{\sigma}^2_1}{f_2 - f_1} \tag{11}$$

with $f_1 = m-2$ and $f_2 = n-m-1$, is *independent* from the *a posteriori* variance of unit weight of the simple model. Hence, for normally distributed observations, the following variance ratios are F-distributed with $m-1$ degrees of freedom in the numerator and $n-m-1$ degrees of freedom in the denominator,

$$F = \frac{\hat{\sigma}^2_1}{\hat{\sigma}^2_{21}} \tag{12}$$

This expression enables us to test the equality of the variances using a two-tailed hypothesis testing procedure at a given *p-level* [2]. When the computed variance ratio exceeds its corresponding theoretical value the null hypothesis is rejected at the given *p-level.*

The application of this testing procedure shows that all the baseline measurements from different antennas (pairwise) are compatible at 0.0001 *p-level.* Hence, we can use all the measurements directly in the composite models.

5. Solutions – Influential Data Analyses

The estimated baseline rates from the simple as well as composite models are given in Table 1. Despite the statistical agreement between the measurements from different antennas, there are large differences between single baseline estimates but the differences are not statistically significant when their errors are considered. One apparent reason for the deviations is the varying number of observations in each solution. Another is a result of more precise baseline measurements in the latter epochs. The composite model solution rates are therefore more in agreement with those single model solution rates that make use of less noisy data. In all cases, the baseline rates

[2]Defined as the statistical significance of a result is an estimated measure of the degree to which it is *true.*

Table 1. Numerical results

Baseline	Estimated Rate (mm/yr)	Nuvel 1A NNR Rate (mm/yr)	$\hat{\sigma}^2$	No. of data
ALGOPARK-NRAO20	-2.3±0.6	0.0	1.55	70
ALGOPARK-NRAO85 3	-0.5±0.4		1.19	48
ALGOPARK-NRAO20+ ALGOPARK-NRAO85	-1.3±0.3		1.36	118
Modified composite solution [3]	-0.1 ±0.4		1.32	117
GILCREEK-NRAO20	-2.7±0.7	0.0	1.96	116
GILCREEK-NRAO85 3	-1.3±0.3		1.53	446
GILCREEK-NRAO20+ GILCREEK-NRAO85	-1.6±0.3		1.63	562
Modified composite solution	None [4]		None	
HARTRAO-KAUAI	-24.2±10.3	-3.0	1.03	13
HARTRAO-KOKEE	-5.2±4.6		1.25	49
HARTRAO-KAUAI+ HARTRAO-KOKEE	-7.4±4.1		1.25	49

from the composite model solutions are not markedly better than those of individual solutions. However, note that composite models include much more data. Figure 4 shows that two of the baseline solutions from single as well as composite models are considerably influenced by a single data point (this is known as *influential data*). The plots show the change in the baseline rate estimate when each data point is sequentially removed from the solution. An influential data point in the ALGOPARK to NRAO85 and NRAO20 composite model solution accounts for 30 percent of the estimated baseline rate (Figure 4a) and there is an almost 50 percent change for another influential point in the HARTRAO to KAUAI and KOKEE composite model solution rate (Figure 4c). Observe that the maximum contributions are located either at the beginning or at the end of single baseline measurements where, by design, they are more effective. The corresponding measurement errors of the influential data are not necessarily the largest errors.

6. Conclusion

The above results show that all the baseline measurements from different antennas are compatible (pairwise) at 0.001 *p-level*. Simple and composite model rates are not markedly different from each other. They are also overall compatible with the NUVEL-1A solution (except HARTRAO-KAUAI baseline solution). Precise baseline measurements dominate the composite model rates as expected. The variances of the composite model rates are not significantly better than those of the simple models. Nevertheless, the resulting series are preferable for further exploratory analyses since they include more data. Composite models are also intuitively more appealing. There are

[3] Modified composite solution is the composite model solution where the influential data point is excluded.
[4] There are no data points in this solution that have a strong influence on the baseline rate.

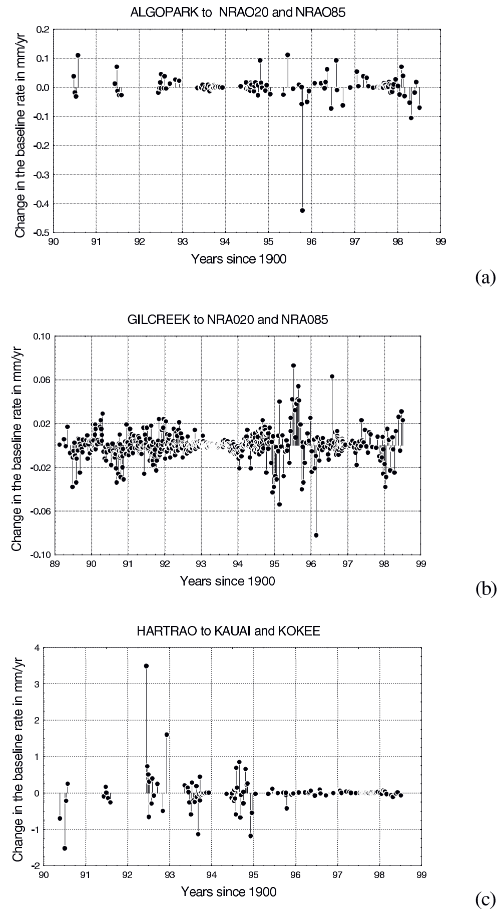

Figure 4. The contribution of each data point to the composite baseline solution rate. Note that the maximum contributions are located either at the beginning or at the end of single baseline measurements. The corresponding measurement errors are not necessarily the largest ones.

data points that influence the estimated baseline rates significantly (up to 50 percent) if they are removed from the solutions. This is especially true for those baselines with fewer observations. Modified composite solutions, where the influential data point is removed, are statistically more reliable since the estimated rates are homogeneously dependent on all data points. For all baselines with few observations and *influential* data points, new observations are strongly encouraged.

7. References

DeMets et al., Effect of recent revisions to the geomagnetic reversal time scale on estimates of current plate motions, Geophys. Res. Lett. Vol. 21 No. 20, p. 2191-2194, 1994. Ma, C., and J. W. Ryan, NASA Space Geodesy Program GSFC DATA Analysis 1998, VLBI Geodetic Results 1979-1998, August, 1998.