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VLBI rates with first order autoregressive disturbances

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Abstract

Repeated Very Long Baseline Interferometry, VLBI, measurements exhibit autocorrelation before the baseline rates are estimated and removed from the data. Autocorrelations reduce markedly once the baseline rates are estimated. This raises the possibility that they may adversely influence the estimated baseline rates if the measurements are in fact autocorrelated but not modeled. We modeled and incorporated autoregressive disturbances into the estimation of baseline rates using an iterative procedure to investigate their presence as well as to assess their impact on the estimated baseline rates and on the corresponding statistics. Our findings indicate that, overall, the estimated baseline rates, their corresponding standard deviations (at one sigma level, based on the *a posteriori* variance of unit weight for each baseline model) are not markedly influenced by the introduction of autocorrelated disturbances. On the other hand, the estimated autocorrelation functions of the residuals show significantly higher order correlations that are due to stochastic and deterministic cyclic effects in the baseline measurements. Corellograms also have signatures of a slippage effect on all baselines that involve HRAS station.

1. Introduction

On the global scale, the geological rates of plate motions are always assumed to be steady in time (DeMets et al., 1990), which has been supported from time to time by the remarkable agreement with the recent plate motion rates inferred from VLBI measurements (McMillan and Ma, 1994). Analysis of earlier VLBI time series showed that baseline measurements are contaminated by systematic effects which are due to the difficulties in modeling the atmospheric delay (Davies et al., 1991). Some of the variations are recognized as quasi-annual variations (McMillan and Ma, 1994) or annual variations (Titov and Yakowleva, 1996), and of unknown nature (Heirtzler and Iz, 1992).

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It is widely known that GPS data is highly correlated in time. Although this correlation is partly due to the intentional degradation of the signal through selective availability, there are other reasons such as atmospheric effects, data processing, instrumental, or orbit errors that may induce such correlations. These effects can be represented with first and second order autocorrelation structure (Iz et al., 1998). Nevertheless, in the case of VLBI, most of the efforts are focused on the sources of errors influencing the measurements and there is no detailed study on the stochastic nature of the time series data.

In this study, we examined modeling the baseline variations to improve the estimated baseline rates and their statistics. We observed that VLBI baseline measurement variations exhibit autocorrelation before the baseline rates are removed from the data that is mostly due to the slow variations induced by plate motions. Once the baseline rates are removed from the baseline measurements, the autocorrelation reduces markedly. This suggest the possibility that, should the measurements be autocorrelated, they may adversely influence the estimated baseline rates and their statistics[1] if they are not incorporated into the model.

Here, we modeled and incorporated autoregressive disturbances into the estimation of baseline rates to investigate their presence and to assess their impact on the estimated baseline rates and on their variances.

VLBI measurements of seven baselines, namely, Gilcreek-Kauai, Gilcreek-NRA085 3, HRAS-Richmond, HRAS-Westford, HRAS-Wettzell, Richmond-Wettzell were considered. This choice was motivated by the fact that these baselines have been observed extensively since 1984 using VLBI, (Ma et al., 1992). Wettzell station is located in Germany (on the Eurasian plate), Richmond station is in Florida, and Ft. Davis (HRAS) station is located in Texas, Westford is in Massachusetts, Gilcreek is in Alaska, Kauai is in Hawaii, and NRAO85 3 station is located in West Virginia.

In the following sections, we first briefly discuss the properties of the baseline measurements and by the current baseline change of rate estimation model. We then calculate the first order autocorrelation coefficient of the baseline measurements before and after the baseline rates are removed. A new statistical model is derived in the subsequent section and the iterative procedure is discussed to estimate baseline rates and the autocorrelation coefficients. We give and discuss the estimates and the corresponding statistics using the new statistical model together with the analysis of the residuals using their estimated autocorrelation function in the final section.

2. Geodetic baselines

It is unfortunate that most of the existing VLBI baselines were neither frequently measured nor measurements span a long period of time. Measurement sessions were primarily designed to verify the plate motions (or stability) as implied by geological and geophysical evidence and to monitor earth orientation and nutation parameters.

[1] It can be shown that if correlations exist but not modeled and introduced into the least-squares estimation process, the variances of the estimated parameters are underestimated (i.e. the estimated variances are larger than the actual variances).

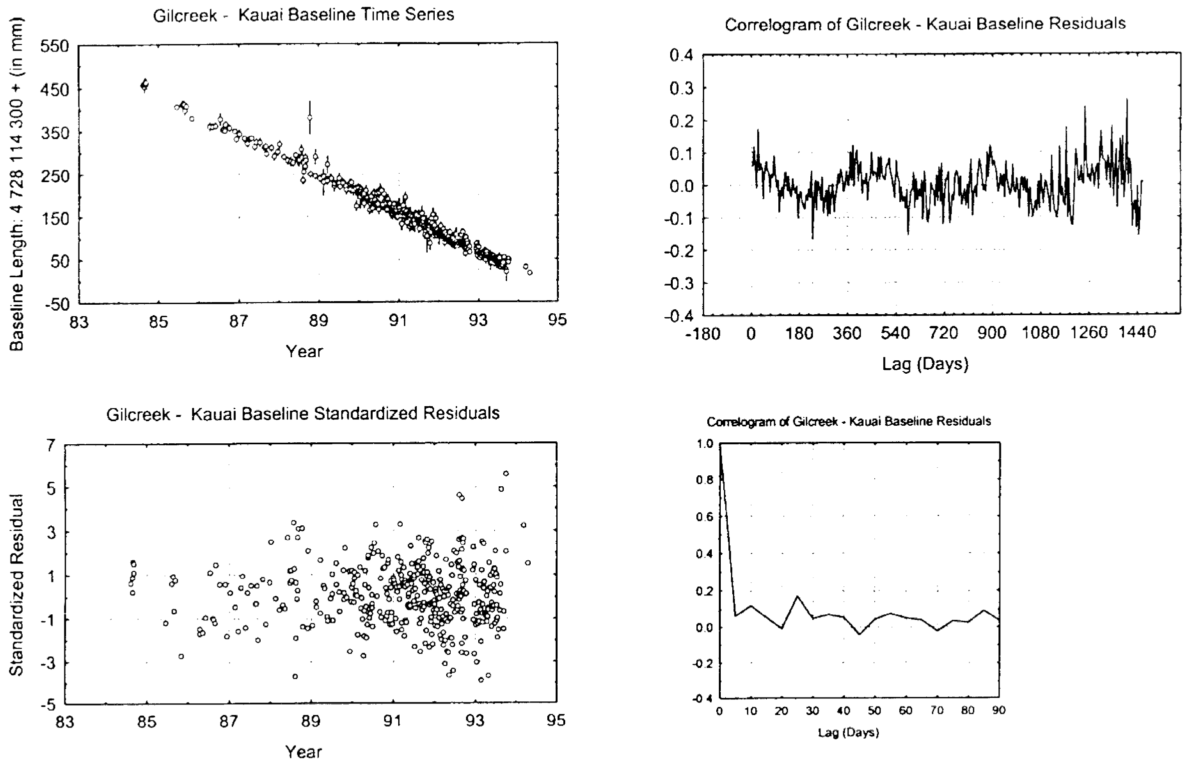

Fig. 1. Gilcreek-Kauai baseline measurements exhibit a weak first order correlation. The estimated residuals however, show the presence of yearly deterministic periodic variations in the observations superimposed with other periodic and non-periodic effects.

We selected baselines that have been continuously monitored. Figures 1–6 show the evolution of the baseline changes in time. The baseline measurements were usually reported at approximately five-day intervals averaged over one-day measurement sessions. There are however deviations from this scenario. Sometimes the measurements are reported more frequently than five-day intervals. For baselines, which include HRAS station, data for more than a year is missing toward the end of the measurement periods.

The data was downloaded from the Internet, which is from the NASA Goddard Space Flight Center's VLBI terrestrial reference frame solution number 1083c, August 1997.[2] The data sets include information about the averaged baseline measurements, the time and an error estimate for each length entry.

3. Estimation of baseline rates from VLBI measurements: the existing model

The adopted approach for estimating baseline rates as reported in Ma et al. (1992) can be described as follows.

[2] http://lupus.gsfc.nasa.gov/vlbi.html

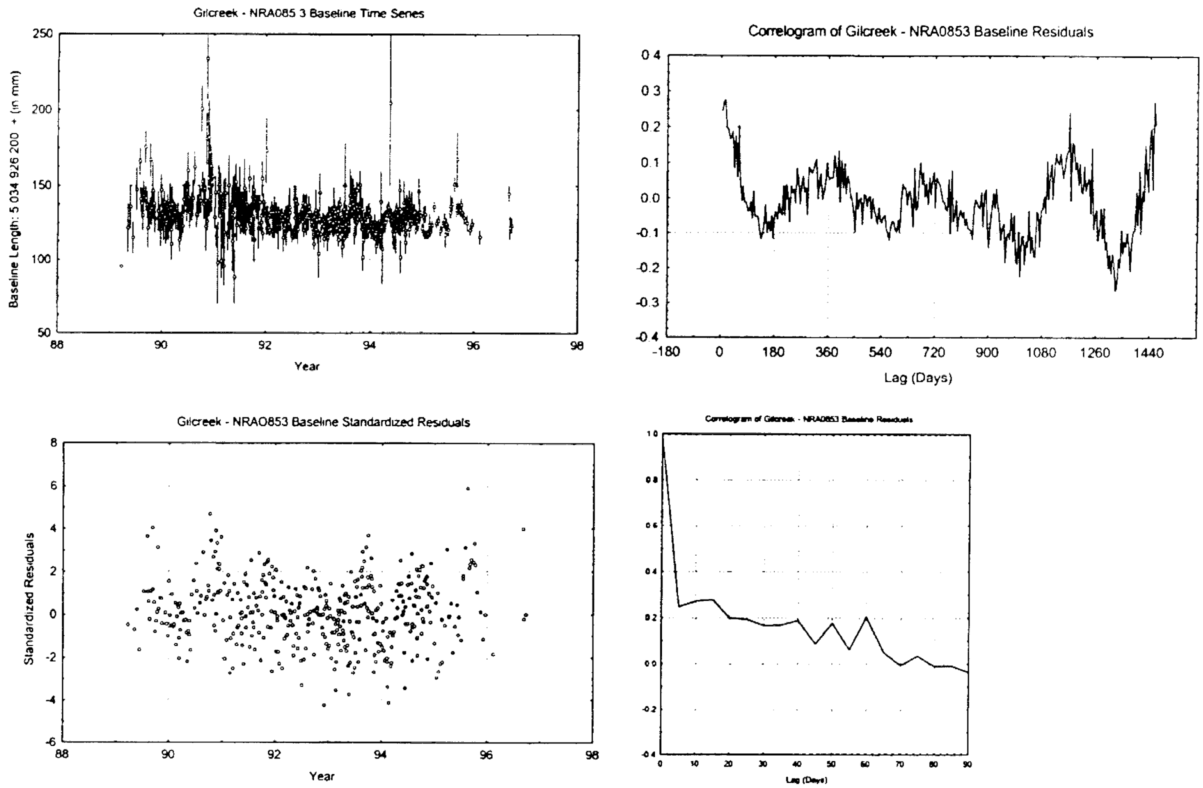

Fig. 2. Gilcreek-NRAO 85 2 baseline measurements exhibit a weak first order correlation but moderately significant autocorrelation effect as revealed by the slope in the plot after the first lag in the 90 day plot. Moreover, the estimated residuals have distinct deterministic signature of a yearly periodic variation. The nature of the short periodic variations requires further study.

Let,

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{u} \tag{1}$$

where, $\mathbf{y}$ is the $n \times 1$ vector of baseline observations, $\mathbf{A}$ is the $n \times 2$ design matrix. The first column of the design matrix is equal to unitary, whereas the second column is composed of the corresponding epoch of observation in modified Julian days divided by 365.25 days. Because of this arrangement, the unknown parameter vector is composed of the intercept and the slope (i.e. the baseline rates per year). The $n \times 1$ vector of disturbances, $\mathbf{u}$, has the following assumed statistical properties,

$$E(\mathbf{u}) = 0, E(\mathbf{u}\mathbf{u}') = \Sigma = diagonal(\sigma_i^2) = \sigma_u^2 \cdot \mathbf{W} \quad i = 1, \cdots, n \tag{2}$$

where, σ^2 is the *a priori* variance of unit weight which is assumed to be unity. This value will be subsequently replaced by the estimated a posteriori variance of unit weight.

A generalized least-squares solution to eq. (1) considering eq. (2) is well-known and is given by,

$$\hat{\mathbf{x}} = (\mathbf{A}'\mathbf{W}^{-1}\mathbf{A})^{-1}\mathbf{A}'\mathbf{W}^{-1}\mathbf{y} \tag{3}$$

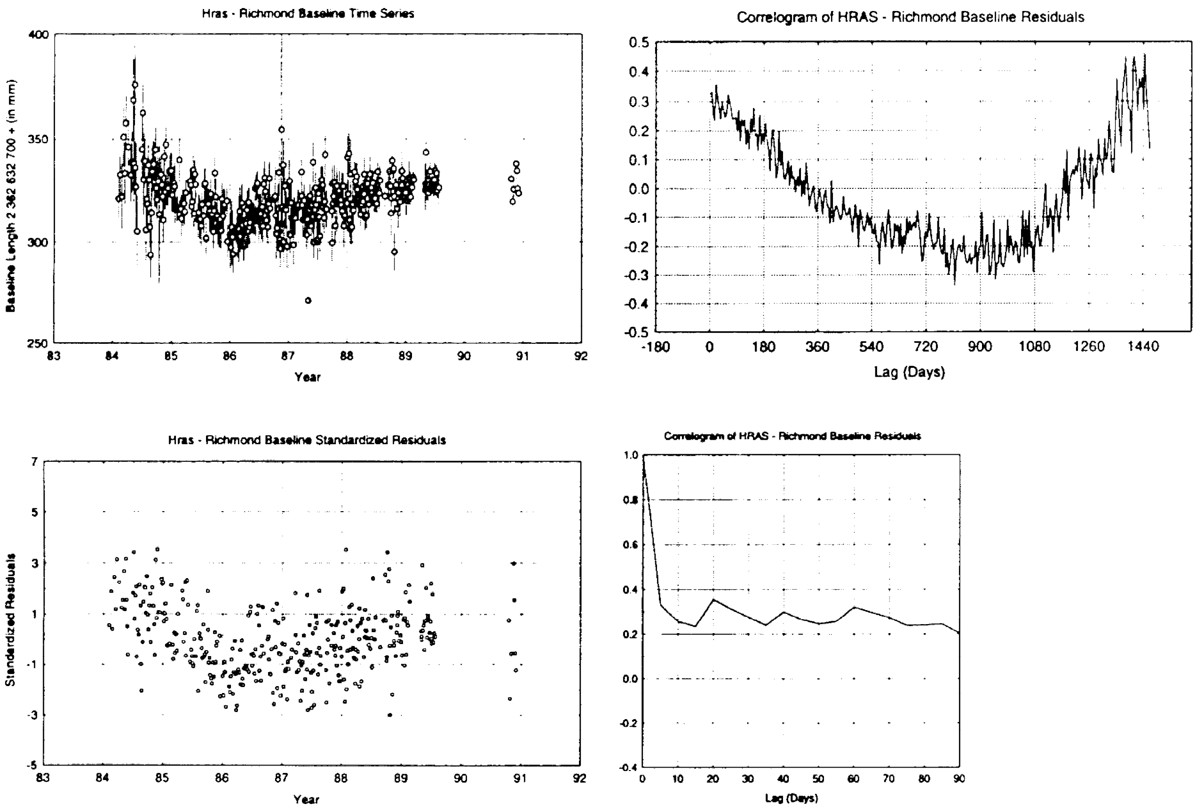

Fig. 3. HRAS-Richmond baseline measurements exhibit a weak first order correlation accompanied with weak autocorrelation. The dominant concave characteristic of the estimated autocorrelation function can be decomposed into a linearly decreasing initial portion of the lag plot, and an increase in the remaining portion. Such behavior can be due to a slippage (jump) in the measurements.

where, $\hat{\mathbf{x}}$ is the 2×1 vector of the least-square estimate of the unknown parameter vector $\mathbf{x}$ which consists of the intercept and the slope. The estimated variance-covariance matrix of the adjusted parameters are given by,

$$\Sigma_{\hat{u}} = \hat{\sigma}^2(\mathbf{A}'\mathbf{W}^{-1}\mathbf{A})^{-1} \tag{4}$$

where, $\hat{\sigma}^2$ is the *a posteriori* variance of unit weight. It is given by,

$$\hat{\sigma}^2 = \frac{(\mathbf{y}-\mathbf{A}\hat{\mathbf{x}})^T(\mathbf{y}-\mathbf{A}\hat{\mathbf{x}})}{n-2} \tag{5}$$

where, circumflex on the variables indicated that they are estimated quantities. The variance-covariance matrix of the adjusted residuals can be computed from,

$$\Sigma_{\hat{u}} = \hat{\sigma}^2(\mathbf{W}^{-1} - \mathbf{A}(\mathbf{A}^T\mathbf{W}^{-1}\mathbf{A})^{-1}\mathbf{A}^T) \tag{6}$$

Figures 1–7 show the baseline measurements and their errors as reported NASA Goddard Space

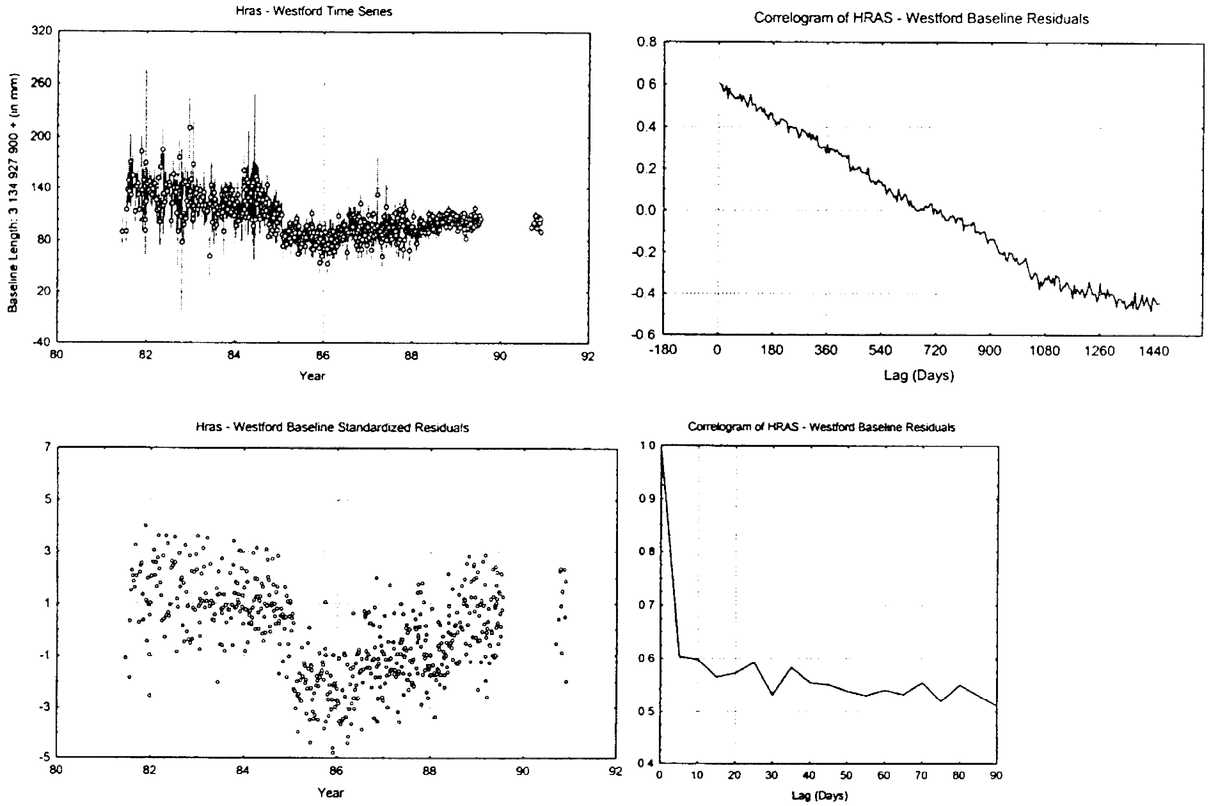

Fig. 4. HRAS-Westford baseline measurements exhibit not only a strong correlation effect which was not removed by the first order modeling (it may be due to a higher order stochastic effect in the data) but also a signature of a slippage (jump) in the measurements as revealed the linearly decreasing nature of the estimated autocorrelation function).

Flight Center's VLBI terrestrial reference frame solution number 1083c, August 1997. We estimated the baseline rates, given in Table 2, using the above model. They are the same as reported in the same solution document. Overall, estimated rates agree well with the geological rates as indicated by the NUVEL-1 model (DeMets et al., 1990).

4. Autocorrelations in the baseline measurements

It is well known in classical least-squares applications that if the observations are correlated but the correlations are not modeled and introduced into the estimation process, the resulting variances of the estimated parameters are underestimated (i.e. they are larger than the true variances). Therefore, the detection and introduction of autocorrelations are needed for more precise error estimates. Moreover, the omission of autocorrelations may adversely influence the estimates themselves. Although the baseline rates are deterministic in nature and hence their estimates should not be influenced by stochastic error considerations, their statistics may change.

Table 1 shows the estimated correlation coefficients for all the baselines considered in this study.

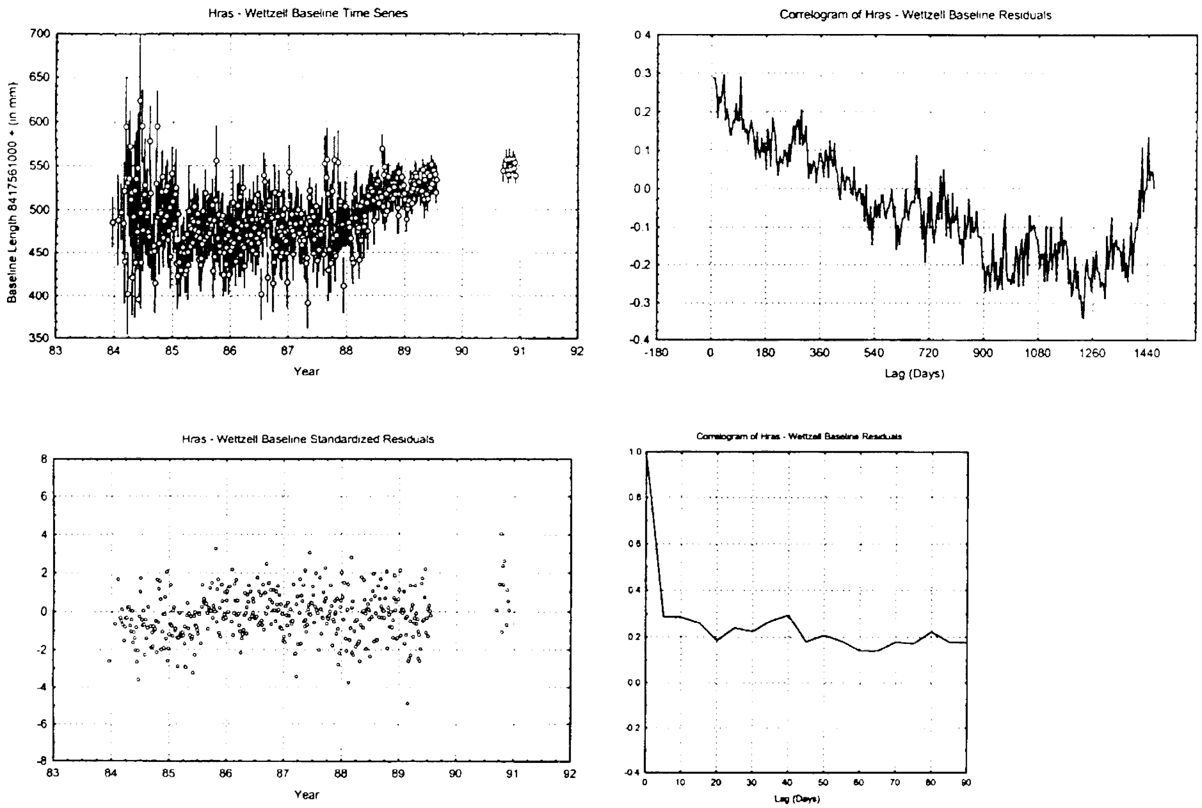

Fig. 5. HRAS-Wettzell baseline measurements exhibit a weak first order correlation accompanied with a jump in the series (again revealed by the linear decrease in the estimated autocorrelation function). The superimposed complex variations in the autocorrelation function are not clear enough to reveal whether the series are contaminated by periodic, almost periodic, or stochastic variations.

The first set of correlation coefficients was calculated from the baseline measurement residuals obtained by removing only the average of each baseline. This process leaves the baseline rates in the measurements. In the second set, the correlations were calculated from the residuals of the baseline measurements after the removal of the trend (slope and intercept) discussed in the previous section. In both cases, the residuals were standardized by dividing each one of them to the corresponding standard deviation of baseline observation.

These results indicate significant reduction in the first order autocorrelations due to removal of the influence plate motions on the baseline measurements. However, once the baseline rates are removed from the baseline measurements, the autocorrelations reduce markedly. At this point, we question that if the measurements are autocorrelated, would they adversely influence the estimation of baseline rates and their statistics?

In order to investigate this premise a first order autoregressive process (also known as linear Markov process) is proposed and incorporated into the estimation. We will first derive a variant of the well-known approach on the first order autoregressive processes—also known as linear

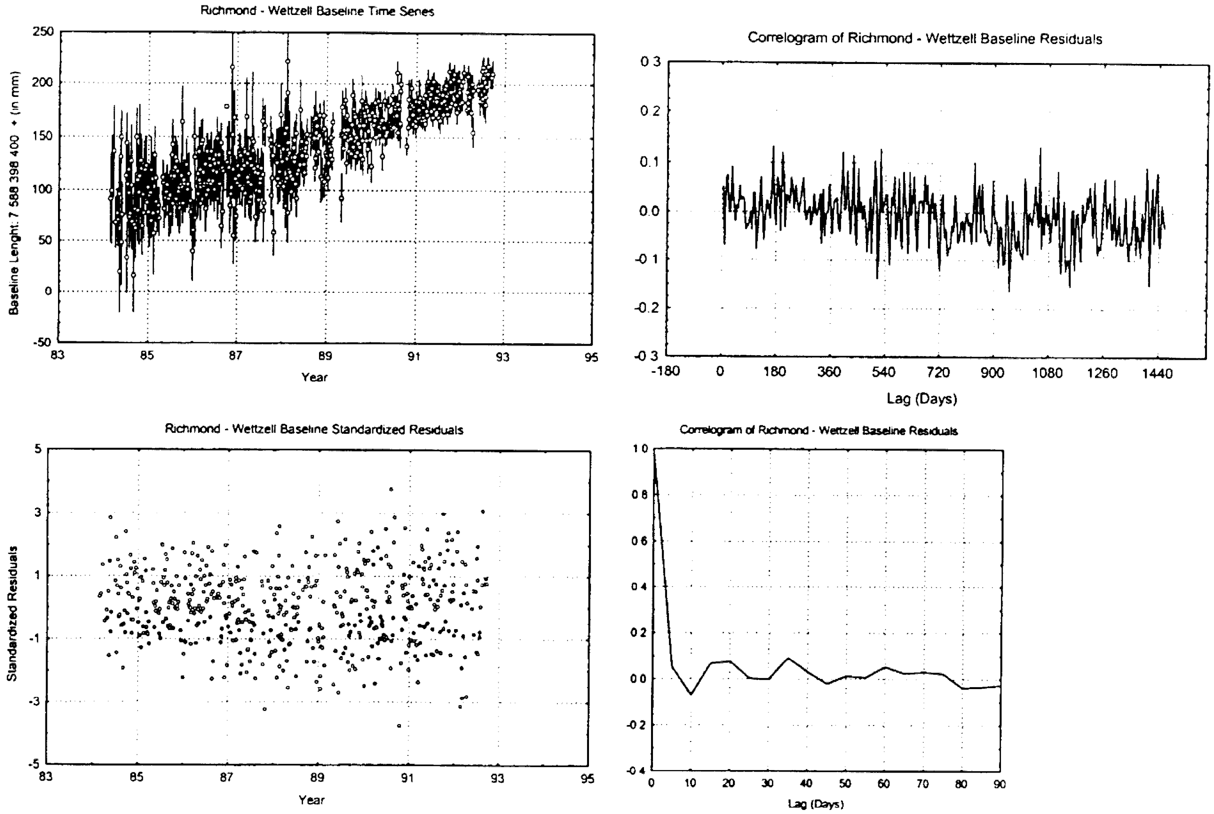

Fig. 6. Richmond-Wettzell baseline measurements exhibit a very weak first order correlation. The overall nature of the auto-correlation function does not reveal a dominant large periodic effect but complex periodic variations.

Table 1
Estimated autocorrelation coefficients before and after the baseline rates are removed

Baseline	Autocorrelation coefficient	
Gilcreek-Kauai	0.865	0.062
Gilcreek-NRA0853	0.321	0.246
HRAS-Richmond	0.410	0.333
HRAS-Westford	0.653	0.594
HRAS-Wettzell	0.698	0.288
Richmond-Wettzell	0.816	0.052
Westford-Wettzell	0.889	0.273

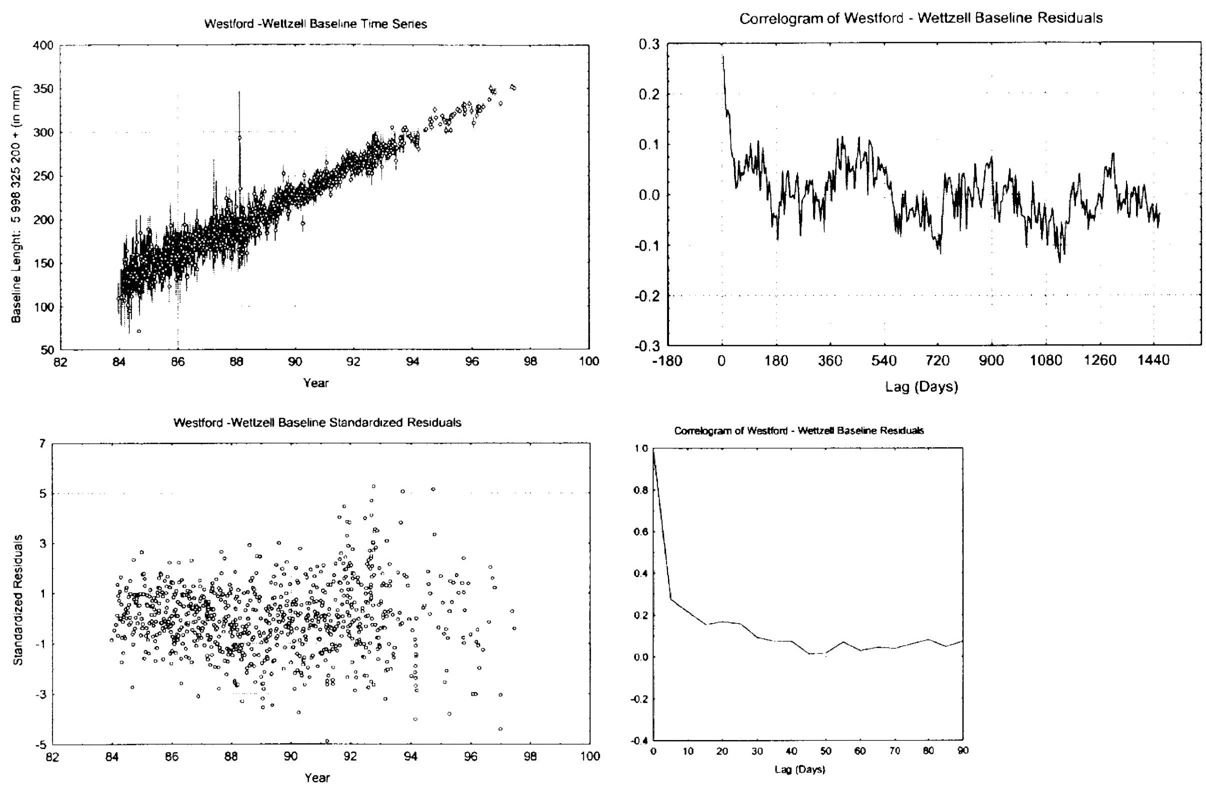

Fig. 7. Westford-Wettzell baseline measurements exhibit a weak first order correlation. The overall nature of the autocorrelation function does not reveal a dominant periodic effect but complex periodic variations.

Markov process, (Kendall, 1994), Tautenburg (1982), Freund and Minton (1979), to accommodate the existing standard deviations of the baseline measurements.

5. An alternative statistical model with autoregressive disturbances

An alternative statistical model for the baseline observations can be described as,

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{u} \tag{7}$$

where $\mathbf{y}$ is the $n \times 1$ vector of baseline observations, and $\mathbf{A}$ is the $n \times 2$ design matrix. The first column of the design matrix is equal to unitary whereas the second column is composed of the corresponding epoch of observation. The $n \times 1$ vector of disturbances, $\mathbf{u}$, is now assumed to be autocorrelated. A first order autoregressive may be expressed as,

$$u_t = \rho u_{t-1} + v_t \quad t = \ldots, -2, -1, 0, 1, 2, \ldots \tag{8}$$

where $-1 \leqslant \rho \leqslant 1$ is the unknown autocorrelation coefficient. It is assumed that the stochastic process, $\{v_t\}$, has the following distributional properties,

$$E(v_t) = 0, \quad E(v_t^2) = \sigma_v^2, \quad E(v_t v_t{}') = 0, \quad (t \neq t') \rightarrow \Sigma_v = \text{diagonal}\,(\sigma_{v_t}^2) \tag{9}$$

To facilitate subsequent developments and to accommodate the non-homogeneous nature of the sample data to be used later, the statistical model defined through eqs. (7–10) will be standardized.

Let,

$$\bar{\mathbf{y}} = \bar{\mathbf{A}}\mathbf{x} + \bar{\mathbf{u}} \tag{10}$$

such that

$$\bar{\mathbf{y}} = \Sigma^{-(1/2)}\mathbf{y}, \bar{\mathbf{A}} = \Sigma^{-(1/2)}\mathbf{A}, \bar{\mathbf{u}} = \Sigma^{-(1/2)}\mathbf{u} \tag{11}$$

Equation (8) can now be expressed as,

$$\bar{u}_t = \rho \bar{u}_{t-1} + \bar{v}_t \quad t = \ldots, -2, -1, 0, 1, 2, \ldots \tag{12}$$

where,

$$\bar{u}_t = \frac{u_t}{\sigma_{v_t}}, \quad \bar{v}_t = \frac{v_t}{\sigma_{v_t}} \tag{13}$$

An equivalent expression for eq. (12) is,

$$\bar{u}_t = \sum_{s=0}^{\infty} \rho^s \bar{v}_{t-s} \tag{14}$$

Considering eqs. (12–14) and eq. (8), eq. (9) and the properties of the geometric series, it can be shown that,

$$E(\bar{u}_t) = 0, \quad var(\bar{u}_t) = E(\bar{u}_t^2) = \frac{1}{1-\rho^2} = \sigma^2 \tag{15}$$

Note that, now, eq. (15) is independent of $\sigma^2_{\varepsilon_t}$. For a sample size T, $\mathbf{u}' = (u_1,\cdots,u_T)$, after some manipulations, the variance-covariance matrix of the disturbances can be written as

$$\Sigma_{\bar{u}} = \sigma^2 \cdot \begin{bmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{T-1} \\ \rho & 1 & & \cdots & \rho^{T-2} \\ \vdots & \vdots & \vdots & & \vdots \\ \rho^{T-1} & \rho^{T-2} & \rho^{T-3} & \cdots & 1 \end{bmatrix} = \sigma^2 \cdot \bar{\mathbf{W}} \tag{16}$$

The above patterned variance-covariance matrix has an analytical inverse, which is given by,

$$\Sigma_{\bar{u}}^{-1} = \sigma^{-2}\bar{\mathbf{W}}^{-1} = \begin{bmatrix} 1 & -\rho & 0 & \cdots & 0 & 0 \\ -\rho & 1+\rho^2 & -\rho & \cdots & 0 & 0 \\ 0 & -\rho & 1+\rho^2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1+\rho^2 & -\rho \\ 0 & 0 & 0 & \cdots & -\rho & 1 \end{bmatrix} \tag{17}$$

From eq. (16), the correlation between two random variables u_t and $u_{t-\tau}$ is $\sigma^2\rho^\tau$, where τ is the time lag. Note that implicit in these results is the assumption that data is equally spaced in time. The correlation decreases for increasing time lag since $|\rho| < 1$.

Now a generalized least-squares solution to the statistical model described by the eqs. (10 16) is

$$\hat{\mathbf{x}}_a = (\bar{\mathbf{A}}'\bar{\mathbf{W}}^{-1}\bar{\mathbf{A}})^{-1}\bar{\mathbf{A}}'\bar{\mathbf{W}}^{-1}\bar{\mathbf{y}} \tag{18}$$

where, $\hat{\mathbf{x}}_a$ is the 2×1 vector of the least-squares estimate of the unknown parameter $\mathbf{x}$. Subscript a indicates that this is an alternative estimate to $\mathbf{x}$ and overbar denotes that the corresponding quantities are standardized. The estimated variance-covariance matrix of the adjusted parameters is given by,

$$\Sigma_{\hat{a}} = \bar{\hat{\sigma}}^2(\bar{\mathbf{A}}^T\bar{\mathbf{W}}^{-1}\bar{\mathbf{A}})^{-1} \tag{19}$$

where, $\bar{\hat{\sigma}}_2$ is the *a posteriori* variance of unit weight. It is given by

$$\bar{\hat{\sigma}}^2 = \frac{(\bar{\mathbf{y}} - \bar{\mathbf{A}}\hat{\mathbf{x}}_a)^T(\bar{\mathbf{y}} - \bar{\mathbf{A}}\hat{\mathbf{x}}_a)}{n-2} \tag{20}$$

Similarly, the variance-covariance matrix for the adjusted residuals is computed from,

$$\Sigma\hat{\mathrm{u}} = \bar{\hat{\sigma}}^2(\bar{\mathbf{W}}^{-1} - \bar{\mathbf{A}}(\bar{\mathbf{A}}^T\bar{\mathbf{W}}^{-1}\bar{\mathbf{A}})^{-1}\bar{\mathbf{A}}^T \tag{21}$$

For the above solution, knowledge about the autocorrelation coefficient is necessary. A starting value can be obtained using the residuals of the first statistical model from eq. (12) by regressing ρ on the residuals or directly from the definition of autocorrelation,

Table 2
Estimated baseline rates with their standard deviations (at one sigma level, based on the a posteriori variance of unit weight). The *a posteriori* variance of unit weight for each baseline model is obtained from weighted (uncorrelated) and weighted and autoregressively correlated least-squares solutions

Baseline	No. of measurements	Estimated rate and its standard deviation (mm/yr.). Weighted solution	Estimated rate and its standard deviation (mm/yr.). Solution with autoregressive errors	*A posteriori* variance of unit weight. Weighted vs. autoregressive solution	Correlation coefficient. Weighted vs. autoregressive solution.
Gilcreek-Kauai	407	−45.86 (0.23)	−45.79 (0.24)	1.43 (1.43)	0.062 (0.062)
Gilcreek-NRA0853	446	−1.30 (0.27)	−1.27 (0.34)	1.52 (1.52)	0.243 (0.245)
HRAS-Richmond	350	1.86 (0.29)	1.81 (0.42)	1.34 (1.34)	0.333 (0.333)
HRAS-Westford	603	−1.43 (0.25)	−1.28 (0.53)	1.75 (1.76)	0.594 (0.604)
HRAS-Wettzell	415	14.05 (0.68)	14.00 (0.94)	1.17 (1.17)	0.288 (0.288)
Richmond-Wettzell	538	14.26 (0.32)	14.25 (0.35)	1.19 (1.19)	0.052 (0.052)
Westford-Wettzell	850	16.87 (0.11)	16.90 (0.15)	1.44 (1.45)	0.273 (0.275)

$$\hat{\rho} = \frac{\sum_{t=2}^{T} \bar{u}_{t-1}\bar{u}_t}{\sum_{t=2}^{T} \bar{u}_{t-1}^2} \tag{22}$$

This two step procedure can be repeated until stable values for the autocorrelation coefficient as well as for the other estimated parameters are obtained. The consistency of this two-stage approach under some general conditions for the case where the stochastic process disturbances, $\{v_t\}$, are assumed to be homogeneous is given in Schönfeld (1969).

6. Numerical results and conclusion

The estimated baseline rates using the new model and their uncertainties at one sigma level are tabulated in Table 2. The first order autocorrelation coefficients for all the baselines are iteratively computed. In all cases, the estimates have converged in the third iteration. Table 2 also shows the estimates obtained using only the measurement errors and the computed autocorrelation coefficients for each baseline.[3]

[3] Some baselines are not regularly observed with five days intervals. They include data that are more frequent. We also solved the rates using the modified baseline measurements with only five-day intervals. No significant changes were observed in the solutions. All the baselines that include an HRAS station have a large gap toward the end of the time series. We repeated the computations excluding this gap and the subsequent data. Again, we observed no significant changes observed in the results.

These results indicate that the estimated baseline rates are practically the same for all baselines because the impact of the correlation structure is expected to be on the statistics of the estimated quantities rather than the estimates themselves. Nevertheless, the standard deviations for the estimates did not change either. The first order autoregressive error structure did not improve any of the baseline models (at one sigma level, and based on the *a posteriori* variance of unit weight). In most cases, the estimated standard deviations are worst than the uncorrelated model. The *a posteriori* variance of unit weight for each solution shows that the new model does not contribute significantly to improving the residuals. The estimated correlation coefficients are all relatively small in magnitude.[4] At this point, we are tempted to generalize and claim that the new model has no value for improving the baseline rates. The rule of parsimony favors the simpler weighted and uncorrelated model. Nevertheless, these results do not rule out the presence of higher order correlations in the measurements. In this case, more insight can be obtained on the nature of baseline variations if we consider the above steps as if they were for pre-whitening of the baseline measurements and examine the baseline residuals after the trend removal.

Because the trend removal (intercept and slope) has been done using the above formal procedure, the autocorrelation function for the estimated residuals can be calculated using the following well known relationship which assumes that the expected values of the residuals are zero,

$$\hat{\rho}_k = \frac{\dfrac{1}{N-k}\sum_{t=1}^{N-k} \bar{u}_t\bar{u}_{t+k}}{\dfrac{1}{N}\sum_{t=1}^{N-k} \bar{u}_t^2} \tag{23}$$

In this expression, k is the running index for the lag of five days, and N is the total number of lags. For $k = 0$, eq. (23) reduces to eq. (22). We use this expression to generate the correlograms of the estimated residuals given in Figs. 1–7. Note that the residuals are standardized, i.e. each residual is divided by the corresponding baseline measurement error. We obtained the same results whether we use the residuals of the weighted or weighted and autocorrelated model. Figures 1–7 also display the standardized residuals. We exhibited correlograms in two different plots: the top plot is for emphasizing the properties of the residuals for lags reaching 1440 days, whereas the bottom plot is for examining the short-term correlations in the data. We have not extended the lags beyond 1440 days since the reliability of the estimated correlations decreases quickly for larger lags.

Figure 1 shows the residual properties of the Gilcreek-Kauai baseline. The residuals exhibit weak first order correlation but have pronounced signatures of yearly variations that were quantified by Titov and Yakowleva (1996). These variations are also superimposed by short periodic variations that can be deterministic or stochastic in nature. These annual signatures are also present in the Gilcreek-NRA085 3 baseline residuals. Correlograms of both baselines (having common Kauai

[4] We should emphasize the ambiguity built in the term *relatively small*. Although data correlation about 0.2–0.3, as a rule of thumb, can be considered as weak for a linear multivariate model, it may be significant in revealing some of the properties of the time series data in an autocorrelation analysis.

station) are very similar. The trend in the ninety-day lag plot is partly due to the periodic effect[5] compounded by the first order correlation which is larger than the previous baseline's.

The next three baselines that involve HRAS station also exhibit some common properties. Up to lag 900, their correlograms are dominated by slowly decreasing lines due to a change of state in the behavior of the baseline measurement errors. The correlogram of the HRAS-Westford baseline is very typical of a case when there is a slippage (jump) in the middle of the data set. Although a similar trend can be due to a moving average effect, we can safely eliminate this possibility. A moving average scheme will create a first order autocorrelation which we have already modeled and estimated. There is also no evidence that the data was processed this way. Furthermore, such an effect will quickly disappear at increasing lags. This baseline also has a large correlation coefficient which is not removed by the first order autocorrelation model we used in the linear model. The upward increase in the HRAS-Richmond baseline correlogram after lag 900 can be explained by a slow trend in the data after the slippage. Although it is possible to derive the correlogram of these scenarios theoretically, similar results can be quickly replicated by simulation. HRAS-Richmond baseline correlogram can be approximated by a data set which is generated by a white noise process in the first portion of the set followed be a slippage and a slow trend superimposed with a white noise process again.

By looking at the downward trend in the baseline measurements for the first two years of HRAS-Richmond baseline measurements, one might argue that there is steep downward trend in the baseline measurements (-14.7 ± 2.0 mm to be exact). However, the error bars and the standardized residuals show that the measurements for the first year are very noisy (they indicate -9.8 ± 8.0 mm rate for this period). Moreover, the above reported estimate for the first period can be misleading since the intercept and slope are inherently highly correlated. This means that any small change in the intercept value will significantly influence the slope of the measurements. Therefore, we can not justify the presence of a downward trend in the first part of the data set with high confidence.

Note that Titov and Yakowleva (1996) reports a yearly signature of amplitude 7.9 mm for the HRAS-Wettzell baseline (also calculated in Heirtzler and Iz, 1992 with a longer period in an open series approach). However, yearly signatures are smaller in HRAS-Westford (of amplitude 3.1 ± 0.6 mm) and much smaller in the HRAS-Richmond baselines (1.0 ± 0.8 mm). In other words, there are no commonalties on all three baseline errors despite the fact that they all share the same station. On the other hand, autocorrelation functions show a slippage effect on all baselines. If there is a change in the state of the baseline measurement errors due to a slippage, harmonic analysis results can be aliased and misleading.

HRAS-Richmond and HRAS-Wettzell baselines are also contaminated by other variations that can be due to a higher order autocorrelation as well as short periodic effects. Correlograms have no clear indications on the nature of these variations.

Lastly, Fig. 6 and Fig. 7 display the residual information of the Richmond-Wettzell and Westford-Wettzell baselines. Both baseline residuals have weak first order correlations superimposed by complex periodic variations. Although these figures do not reveal any peculiarities whether the variations are cyclic or not (higher order autocorrelations may have the same signatures), the

[5] Note that the correlogram of a sine or cosine function is always a cosine function independent of the phase of the function.

amplitudes of the annual variations, 3.6 mm ± 0.9 mm and 5.5mm ± 0.4 mm respectively, as reported by Titov and Yakowleva (1996) support the presence of periodic effects.

Abstract

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